On the Macroscopic Origins and Consequences of Economic Inequality: An Evolutionary Perspective*

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Abstract

Income inequality can vary rather dramatically across societies. While in some countries the average income of the richest 10% does not exceed 5 or 6 times that of the poorest 10%, in others the same ratio can reach up to 90 or 100. Moreover, such differences can persist and even increase over long periods of time. In order to address such facts, we develop a theory, based on socio-cultural evolution, that highlights the stability and heterogeneity of a society's economic environment as a fundamental source of long-term inequality. We show that steady and diverse economic environments provide a selective advantage for cooperative or mutualistic behavior, thereby generating economic *equality*, whereas fluctuating and singular ones favor selfish behavior, thereby inducing economic *inequality*. We also show that more equal societies exhibit a higher degree of income and social mobility and are more resilient and robust in the sense of being quicker to recover from shocks and to return to normalcy than unequal ones. We thus provide a rationale for the emergence of inequality, its persistence and negative correlation with income and social mobility, and highlight its role in determining the fragility and robustness of a society. Recent empirical evidence for our main results and policy implications to promote cooperation and equality are also briefly discussed.

Keywords: Income inequality; social mobility; macroeconomic volatility; fragility; cultural evolution; cooperation; evolutionary entropy. *JEL Classification*: E24, E32, O11, O13, O15

1 Introduction

The question of why some societies are wealthier and/or exhibit stronger growth than others has received much attention within economics, as has the question of possible consequences of income inequality for development and growth (Acemoglu [1], Aghion et al. [7], Bénabou [19], Galor [49], Galor and Zeira [51], Ray [70]; see also Lucas [63] for a reduced form model). Somewhat less studied is the question of why

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societies differ so much in terms of inequality; and why such different levels of inequality persist for so long across societies (Acemoglu and Robinson [5], Atkinson [15], Atkinson and Bourguignon [16], Boix [23], Piketty [67]). We here introduce an empirically grounded, evolutionary framework that provides an explanation for why some societies are stratified, economically fragile, and characterized by inertia of social mobility, while others are highly egalitarian, resilient towards shocks, and exhibit high levels of intergenerational mobility. An advantage of our approach is that it allows, within a single framework, to account for three critical dimensions of the problem, namely, the spread of equality/inequality, the correlation between income inequality and social mobility across generations, and the social and economic instability of highly stratified societies. To the best of our knowledge this has not been addressed so far in a single or unified framework in the economics literature. The evolutionary mechanism we propose acts alongside the more explicit economic and political channels studied in the literature.

Inequality has traditionally been addressed in economics in terms of supply and demand for different types of factor endowments (Atkinson and Bourguignon [16] contains a survey). In these models, economic inequality is derived from the differences of remuneration for the different factors. These models are typically formulated within the context of (competitive) markets and are sufficiently flexible to provide insight into a variety of empirical aspects of inequality, including the explanation of a widening gap between skilled and unskilled labor due to technological change. An important limitation, however, is that they take as given the distribution of factor endowments, as well as the pricing mechanism that ultimately establishes the compensation of the different factors. Not surprisingly, they are unable to explain two empirical observations that set apart stratified and egalitarian societies, namely, *(i)* the correlation between economic inequality and intergenerational mobility: intergenerational earning mobility is low in countries with high income inequality such as several countries in Latin America, Africa, and the Middle East, and is high in Scandinavian countries where income distribution is less stratified and more egalitarian; *(ii)* the correlation between economic inequality and resilience, that is, the capacity of a society and its economic institutions to maintain a high level of economic and political stability and functionality in spite of internal and external shocks; resilience is characteristic of egalitarian societies such as the Scandinavian countries; the social and economic fragility of stratified societies is strikingly illustrated by the recent social and economic disruptions observed in Libya, Tunisia, and Egypt.

More recently, the economics literature on inequality has also addressed questions of persistent inequality and intergenerational mobility (Piketty [67] contains a survey; Corak [31] also discusses more recent empirical evidence). These have helped to elucidate the shortcomings of the supply and demand model and have delineated several socio-cultural factors which contribute to the origin and persistence of inequality. These include: *(i)* the transmission of wealth from parents to children through inheritance, which can amplify economic inequality and facilitate its persistence across generations; *(ii)* the intergenerational transmission of ambition and the recognition of economic success and social prestige; underlying this is the hypothesis that individuals tend to compare their social achievements to the reference groups to which they belong, such that individuals with lower class origin are less motivated than individuals with upper class origins to make human capital investments that may enhance their social status; *(iii)* the statistical discrimination or the prevalence of self-fulfilling discriminatory beliefs, whereby persistent generational inequality between two social groups with homogeneous characteristics can be generated by discriminatory hiring policies based on certain assumptions regarding the abilities of the individuals that belong to the different groups.

This socio-cultural and evolutionary perspective resolves certain anomalies of the supply and demand models. But it also has its shortcomings in that it does not account for the fact that these features, the transmission of wealth through inheritance, the intergenerational transmission of ambition and work ethic, and the extent of discrimination, also show a large variation *across* societies.[1] Societies which show significant differences in economic inequality typically differ in terms of their macroeconomic environment, defined in terms of the steadiness and diversity of its resources. Countries with severe economic inequality such as Nigeria, Saudi Arabia, or Venezuela have economies which are dependent on a singular resource, namely oil, whose economic benefits are often non-constant. Countries with little economic inequality, such as the Scandinavian countries have economies based on more diverse sets of resources with relatively more constant benefits. This suggests that the macroeconomic environment may indeed be a critical factor in generating persistent inequality.

The model we develop accounts for this by highlighting certain characteristics of the economic environment as basic driving forces implicated in the redistribution of resources in society. The idea is that modes of behavior, mutualistic or self-serving, are correlated with more or less egalitarian income redistribution, and that the overall macro-economic environment confers certain advantages to different modes of behavior. Intuitvely speaking, when the environment is steady and diverse, cooperative or other-regarding modes will be more effective at exploiting the resources, and many individuals will benefit from such behavior, thus leading to a more egalitarian society. However, when the environment is variable and singular, so, in particular, is subject to episodes of abundance interpersed by periods of scarcity, then self-regarding modes will have an advantage, and this behavior will benefit few individuals, thus leading to a stratified or hierarchical society.

These two types of environments – steady and diverse *versus* variable and singular – are the extremal states which we distinguish in the analytical formulation of our model. Our explanation of the large variation in income inequality from highly stratified to highly egalitarian societies, which is the main object of this paper, is based on an evolutionary analysis of the spread of *cooperation*. The term cooperation generically refers to an engagement with others in activities which confer benefits and generate costs. At the individual level, we distinguish between mutualistic dispositions, where transactions are pursued to the mutual benefit of interacting individuals, and selfish dispositions, where transactions are pursued uniquely to the benefit of the self. In our model, the complete spectrum of cooperative behavior, broadly speaking, – from mutualistic to selfish – has an evolutionary basis and can be understood in terms of the interaction of three processes: *(i) variation*, individuals in a population vary in terms of their propensity to cooperate; the differences in behavior – whether mutualistic or selfish – are largely the adaptive response of individuals to the socio-economic environment in which they operate; *(ii) selection*, different levels of cooperative behavior confer different capacities to appropriate resources from the economic environment and thus provide different economic benefits which are the basis for the selection process; *(iii) transmission and inheritance*, individuals bound by genetic, cultural, or socio-economic ties generally share similar interests and display similar cooperative behavior; these traits are often transmitted through social learning, or simply by means of membership to a group that subscribes to given norms. These three processes, variation, selection and transmission, imply that, over time, the composition of the population will become

[1] Empirical support for differences in the intensity of statistical discrimination between societies is starkly illustrated in a recent study of anti-semitism in Germany over a period of 600 years by Voigtländer and Voth [82]. These authors show that discrimination against Jews was highly correlated with the economic environments; in particular, discrimination was lower and less persistent in regions with high levels of trade, (which we take to represent both more constant and diverse environments). In particular, it is consistent with the idea that the external economic environment is a relevant force for the emergence of dispositions, cooperative or self-serving, that also determines the level of discriminatory behavior and the spread of economic inequality. Notice that inequality differences measured by different levels of concentration of landownership in Germany and Prussia documented by Ziblatt [85, 86] are also consistent with our theory.

increasingly characterized by individuals whose cooperative behavior will be reflective of the underlying socio-economic environment. The kinematics of the socio-cultural evolutionary processes we study necessarily depends on the capacity of individuals to exploit the resources of the economic environment and to reinvest these for their own benefit and for the benefit and persistence of their socio-cultural affiliation. In Section 7 we discuss empirical evidence that indeed finds a negative corelation between trust (as a proxy measure for cooperation) and macroeconomic volatility.

At a formal level, our analysis revolves around a statistical measure of overall interaction called *evolutionary entropy*. This quantity is a statistical measure of the cooperative dispositions – whether mutualistic or self-serving – of the social groups or individuals in the society. It describes the extent to which individual groups in a society share and allocate resources between each other and measures the number of accessible pathways of income or commodity flows between individuals in the network. A *low entropy* social network is described by a poverty of pathways of flows; such a network is characterized by limited interactions between groups or by a single group of individuals with more pathways being directed towards it. A *high entropy* network is described by a diversity of interactions and by a large number of pathways between individuals in the different groups. Demetrius and Gundlach [39] show that both the level of cooperation and the capacity to appropriate resources can be analytically described in terms of the concept of evolutionary entropy.

The significance of the evolutionary entropy concept towards our understanding of the origin of stratified or egalitarian societies resides in its relation to certain key variables, besides the level of cooperation mentioned above. In this paper, we formally derive relationships between the evolutionary entropy measure and the following three variables:

- *Inequality:* Evolutionary entropy is negatively related with the Theil index, which is a standard, entropy-based measure of income inequality. In this sense, societies with high entropy will have low income inequality, whereas societies with low entropy will have high income inequality.

- *Income and Social Mobility:* Evolutionary entropy is positively related with statistical measures of income mobility and of social mobility. Thus societies with high entropy will exhibit high income and social mobility, whereas societies with low entropy will exhibit low mobility.

- *Resilience:* Evolutionary entropy is positively correlated with the stability or resililence of the social network. Thus our entropy measure also relates to the ability of the society to withstand and react to shocks. Egalitarian societies are highly resilient and resistant to shocks and to disruptions of their basic structure; stratified societies are highly sensitive to such perturbations.

The concept of evolutionary entropy has its origins in the ergodic theory of dynamical systems (Arnold et al. [14], Demetrius [37, 38]). This mathematical concept is the cornerstone of directionality theory (see Demetrius [38] for a recent overview). One of the central tenets of the theory is the *entropic selection principle*. This principle states that the dynamical changes in evolutionary entropy under the process of variation and selection are contingent on the resource abundance and diversity of the environment. These changes can be characterized in terms of the following tenets: When resources are *constant* in abundance and *diverse* in composition, communities with higher evolutionary entropy will have a selective advantage and increase in frequency; when resources are *fluctuating* in abundance and *singular* in composition, communities with low entropy will have a selective advantage and will increase in frequency.

We exploit the entropic selection principle – transposed to a macroeconomic setting – to shed light on three key socio-economic phenomena, namely, the large variation in economic inequality and in income and social mobility across countries, and the fragility or robustness of societies, which have hitherto seemed untractable within one framework in the economics literature. The main results we obtain can be qualitatively described as follows.

(1) **Redistributive Selection Theorem.** When the income generation process is steady and based on a diverse set of activities and resources, a cooperative disposition will spread and communities will tend towards more equal economic redistribution; when the income generation process is fluctuating and based on a narrow set of resources and activities, a selfish disposition will have a selective advantage and societies will tend towards less equal economic redistribution.

This provides a new perspective on Adam Smith's invisible hand in relation to inequality. According to Adam Smith, the private pursuit of self-interest would lead *always*, as if by an invisible hand, to the well-being of all. Our analysis indicates that this condition of *well-being of all* will only be achieved in the long run if the environment and external resource conditions are constant and diverse. If the environment is fluctuating and singular beyond a certain threshold, then the invisible hand would lead in the long run to a situation where the *well-being of a few* is achieved.

(2) **Income and Social Mobility Theorem.** Societies with a steady and diverse income generation process will be characterized by high intergenerational income and social mobility; whereas societies with a fluctuating and singular income generating process will be characterized by low mobility.

This constraint on income and social mobility further increases the degree of inequality in the society and may drive society towards greater instability and inefficiency. Thus societies with important income inequality may become increasingly unequal, inflexible, and stagnant. By putting the steadiness of the environment at the center of the analysis the theory provides a new perspective on the long-standing debate in sociology of whether industrialist-capitalist societies tend towards more income and social mobility or towards less (Piketty [67]). From the viewpoint of our theory, the debate has possibly overlooked an important explanatory variable, namely, the steadiness and diversity of the economic environment. At the same time the result also implies a negative correlation between income inequality and income mobility, and thus provides an alternative explanation for what has recently been termed the *"Great Gatsby Curve"* (Corak [31]).[2] Finally, the fragility or instability of unequal societies is reflected in our third main result.

(3) **Perturbation Stability Theorem.** Societies with unequal distribution of resources – highly stratified societies – are inherently unstable and highly sensitive to small perturbations in resource allocation; they take longer to return to normalcy than more equal ones. Equal societies are more stable and robust to pertubations; they are quicker in returning to normalcy than less equal societies.

This captures part of the large inefficiency and instability observed in highly stratified countries, e.g., the recent events in the Middle East (specifically in Egypt, Tunisia, Libya, Syria among others). These can be seen as instances of this theorem, where relatively small shocks can lead to major disruptions affecting the everyday life and organizational structure of an entire country for a prolonged period of time. Again, in Section 7 we briefly discuss some empirical evidence relating the robustness or fragility of countries, generally speaking, to thier degree of inequality.

[2] Several papers, empirical and theoretical, have argued that high social mobility or the "prospect of upward mobility" (POUM) may influence negatively the demand for taxation and redistribution; see, e.g., Bénabou and Ok [20], and also Alesina and Giuliano [9] and Corak [31] for further discussion. In our theory, inequality and immobility always tend to move in the same direction.

Further Related Literature. Within the vast literature on macroeconomics, growth, and inequality, a number of papers have studied the link between inequality and uncertainty or macroeconomic volatility. Some of these papers link inequality to agents' uncertain endowments and different attitudes towards risk, Caroli and García-Peñalosa [28], Checchi and García-Peñalosa [29], or through saving and the labor supply, García-Peñalosa and Turnovsky [52], or directly through various direct or indirect effects of the business cycle that amplify the inequalities, Atkinson and Morelli [17], Stiglitz [77, 78]. These papers examine explicit economic mechanisms that are embedded in relatively complex macro-economic models. Some papers study the link empirically. Breen and García-Peñalosa [27] and Laursen and Mahajan [60] study the relationship across countries, while Huang et al. [55] studies it for the US. Consistent with our theory, these papers find a significant and positive correlation between income inequality and *macroeconomic volatility*, measured as the standard deviation of the rate of growth of real per capita GDP; we come back to this in Section 7.

Also related is the literature that studies the effect of democracy and political institutions on redistribution. Following the seminal paper of Meltzer and Richard [65] it suggests that higher levels of democracy should be associated with higher levels of redistribution (see Acemoglu et al. [4] for a survey and a discussion of empirical evidence; Alesina and Giuliano [9] have a survey on preferences for redistribution). The empirical evidence is rather mixed, particularly on the relation between democracy and inequality. Acemoglu and Robinson [5] discuss more generally the consequences of institutions and their capture by elites on growth and redistribution. To this and the above mentioned literature, our approach adds a novel, complementary mechanism, that works through the evolution of cooperative dispositions among individuals and groups in the society, and that acts as a fundamental force alongside the more specific, economic and political channels studied in the literature. In particular, a society organized as a democracy with a highly fluctuating and singular environment and income process will have an innate tendency towards inequality.

The paper is organized as follows. Section 2 presents the main framework adapted from directionality theory. Here we articulate the basic framework which we use to address the problem of the origin and evolution of income inequality. In Section 3, we present two examples of simple economies to illustrate our approach within familiar economic growth settings. Section 4 contains the main analytical result, the entropic selection principle. Section 5 establishes the relation between evolutionary entropy and a standard measure of inequality, the Theil index, as well as measures of income mobility and of social mobility; it then states the main economic results of the paper. Section 6 addresses the instability of unequal societies and discusses some possible consquences. Section 7 discusses some empirical evidence mostly from the existing literature, and Section 8 concludes. The main proofs and several basic concepts and results from directionality theory, as well as some examples, are contained in the Appendix.

2 The Framework

Our analysis is based on a model of the cultural evolution of a society. The basic building block are individuals that are organized in groups or classes and that interact with each other, within and across classes, in the production and allocation of resources. We distinguish two main processes, one describing the evolution of the population $(N(t))$ and another one describing the evolution of aggregate income or production $(Y(t))$ in the economy. The interaction between the individuals in the society is at the center of our theory and is captured by what we call an interaction matrix A described below. It will determine the society's intertemporal income generating process.

Population and Income. Consider a society with a **total population** $N(t)$ of individuals distributed in d **classes**, written as,

$$N(t) = \sum_{i=1}^{d} n_i(t), \tag{1}$$

where $n_i(t)$ denotes the number of individuals (or households) in class i in period t. The classes may be thought of as describing occupational classes, which we assume thoughout to be of equal size $n_i(t) = N(t)/d$.[3] These individuals engage in several activities in order to produce and exchange commodities and services. The **total income** (or production) $Y(t)$ of the economy,

$$Y(t) = \sum_{i=1}^{d} y_i(t), \tag{2}$$

is the sum of income going to the different classes, where $y_i(t)$ denotes the amount of income going to class i in period t; we also write $y(t) = (y_i(t))$ for the vector of incomes of the different classes. Prices are not modeled explicitly in this set-up. We assume all goods (outputs, inputs, services) are exchanged against a numeraire commodity and it is the transfers of units of this numeraire commodity during the given periods of time that we record here. As is standard with population models (e.g., Dawson [36] and Haccou et al. [53]; see also Demetrius [38]), the discreteness and finiteness of the income and population processes is an important element in the present analysis.

Interaction Matrix and Income Generation Process. The agents in the society are jointly involved in a process of income generation where, besides exchanging and transforming resources or producing goods and services within classes, they also exchange and transform resources across classes, interacting in joint activities of production or consumption, thereby also transfering income, commodities and services to each other. This income generation process is a fundamental characteristic of the society that, as we will see, summarizes its social and economic interactions and determines its long-run development. We describe this process compactly using a type of transition matrix, we refer to as the **interaction matrix**,

$$A = (a_{ij}), \qquad a_{ij} \geq 0, 1 \leq i, j \leq d, \tag{3}$$

where a_{ij} measures the marginal rate of "contribution" of a unit of income in class j in period t towards income in class i in period $t+1$. We assume that all transactions are made in discrete units – "representative" units of the numeraire good – and that the (finite) number of units generated and transfered are random but occur according to the fixed rates a_{ij}, which constitute per period averages.

The interaction matrix A can be thought of as representing a directed graph over d nodes (for the d classes), where $a_{ij} > 0$ corresponds to a directed link from node j to node i of intensity a_{ij}. For simplicity, we assume the matrix is irreducible, meaning that all entries are strictly positive, implying the underlying directed graph is strongly connected; for any pair of nodes i and j there is a directed path going from i to j and from j to i of strictly positive intensity. As we will argue later, the structure of the underlying graph can be seen as reflecting the social preferences of the individuals in the society.

The interaction matrix A is at the center of our analysis. It serves to characterize the society's **in-**

[3] All results go through if this only holds to an approximate degree, $n_i(t) \approx N(t)/d$. We do not elaborate on this.

tertemporal income generation process, that is, the steady state law of motion for aggregate income,[4]

$$y(t+1) = Ay(t). \tag{4}$$

In general one can imagine the economy following a non-linear dynamics of the form, $y(t+1) = A(t)y(t)$, where the matrix $A(t) = (a_{ij}(t))$ is more generally a matrix varying with time and whose entries can also depend on the distribution $y(t)$, where $y(t) = (y_i(t))$ measures the amount of income of the different classes, $i = 1, \ldots, d$ at period t. Our focus in this paper is in the economy once it has evolved to the steady state, such that the entries of the interaction matrix are all fixed and constant. The steady state distribution is then obtained by computing the right eigenvector, $v = (v_1, \ldots, v_d) \in \mathbb{R}^d_+$, corresponding to the dominant (maximal) eigenvalue $\lambda \in \mathbb{R}_+$, such that

$$Av = \lambda v. \tag{5}$$

The magnitude $\bar{v}_i = v_i / \sum_{j=1}^d v_j$ measures the steady state share of aggregate income or product of the class i, $i = 1, \ldots, d$. We also refer to $r = \log \lambda$ as the **growth rate** of the steady state path.[5]

The fact that income is recorded in “representative” units of the numeraire good is essential to our model and its underlying stochastic structure. The idea is that in our model only integer units are ever generated, exchanged or transfered. In particular, the rates (a_{ij}) are averages representing random integer-valued realizations. An agent purchasing at a rate of 0.2 cars per year, will on average buy one car every five years. The logic that the 0.2 is an average while the actual realization is integer-valued and stochastic, carries over to all interactions in both exchange and production. This creates a natural source of randomness in the system that is inherent in economic interactions and that we account for in a stylized way that is consistent with the theory of population processes, where rather than income what is generated and distributed in different classes are individuals of the population.[6] In our economic setting the magnitudes of the “units of GNP” vary greatly depending on the commodities or services exchanged or produced; these can be very large (as with housing, durable goods) or very small (as with water of flour). We assume there is an average or “representitative” such unit which is what we assume at the basis of all transactions in our stylized model. This will be important when deriving a stochastic process form the steady state law of motion representation given by Equation (4).

[4] The balanced growth model of Solow and Samuelson [75] is perhaps the closest to our present approach. They consider a productive environment of a closed economy for which “outputs become inputs one unit of time later” so that they formulate the intertemporal relation between quantities of d commodities (inputs and outputs) produced in period $t+1$ with the quantities of the same commodities produced in period t. This gives rise “casual system of nonlinear difference equations,”

$$y_i(t+1) = H_i(y_1(t), \ldots, y_d(t)),\ i = 1, \ldots, d,$$

or $y(t+1) = H(y(t))$ for short. However, if one assumes that the map H is linear, the system reduces to,

$$y_i(t+1) = h_{i1}y_1(t) + \ldots + h_{id}y_dt, \text{ for } i = 1, \ldots, d,$$

or $y(t+1) = Hy(t)$ for short. The coefficients $h_{i1}, \ldots, h_{id} \in \mathbb{R}_+$ can then be interpreted as marginal productivities of the different commodities produced in period t in the production of commodity i produced in period $t+1$. Our framework shares with this approach the way of modeling an intertemporal production process. An important difference is that we also incorporate an allocative dimension in the sense that, in our model, $y_i(t)$ is income held by class i, and what we describe represents the *society*'s income generation process rather than a pure production process. Our matrix A describes the intertemporal relationship between different amounts of income distributed across the d different classes in period t and the income in the same classes in period $t+1$.

[5] Thus, if at time 0 we have income levels $v(0) = (v_1(0), \ldots, v_d(0)) \in \mathbb{R}^d_+$, then for any time $t > 0$, we expect, $\mathbb{E}_0[v(t)] = \lambda^t v(0) = (\lambda^t v_1(0), \ldots, \lambda^t v_d(0)) = (e^{rt}v_1(0), \ldots, e^{rt}v_d(0)) = e^{rt}v(0)$. The income levels in the different classes are implicitly assumed to grow at the same rate in the steady state.

[6] A particularly classical model is the one using the *Leslie matrix* (e.g., Demetrius [38] and Demetrius and Gundlach [39]) to describe the evolution of the population and the distribution of individuals in different age-classes. The entries in that matrix correspond to reproduction and survival rates and the natural “units of account” are individuals. In Appendix A.1 we briefly present this example.

Evolutionary entropy. From the interaction matrix A we can derive the **Markov matrix**

$$P = (p_{ij}) = \left(\frac{a_{ij} v_j}{\lambda v_i} \right), \quad 0 \le p_{ij} \le 1, 1 \le i, j \le d.$$

describing the probabilities with which "representative" units of income are generated or transfered across classes. An element $p_{ij} (\ge 0)$ can be interpreted as the probability that, in the steady state, a "representative" unit of an individual in class i originates from a "representative" unit from an individual in class j in the previous period.[7] Define $\pi = (\pi_1, \ldots, \pi_d)$ as the corresponding stationary distribution satisfying $\pi P = \pi$, where an element $\pi_i (\ge 0)$ reflects the probability (in the sense of fraction of time) that a given unit of income is with individuals in class i. Then we can define the **evolutionary entropy** of the society as

$$H = -\sum_{i=1}^{d} \pi_i \sum_{j=1}^{d} p_{ij} \log p_{ij}, \quad 0 \le H \le \log d, \tag{6}$$

where $H = 0$ indicates minimal, and $H = \log d$ maximal, entropy. This is the central concept of the present approach.[8]

The concept of evolutionary entropy describes the number and intensity of pathways of income flows between the individuals in the social network. It is a measure of the strength of the interactions of the productive and allocative process between the individuals in society.[9] The higher the number of links or flows between agents within and across classes the larger the entropy. As we will see in Theorem 4, it is also a measure of stability of the society in that it is a measure of the rate at which the society returns to the steady state after a shock.

Interaction matrices coresponding to an egalitarian and a stratified society are represented in the two graphs of Figure 1. In the left-hand graph there are many distinct pathways of resource flows, thus describing an egalitarian or high entropy interaction network. In the right-hand graph the pathways are essentially directed towards a single group, (here $a \gg \epsilon > 0$ and $\epsilon \approx 0$), thus describing a sratified or low entropy network.

Fluctuations around the Steady State. Given the steady state law of motion described by (4) and the discreteness and finiteness of the units transacted, the actual aggregate production $Y(t)$ fluctuates around the steady state.[10] In the Appendix we sketch the underlying probabilistic structure derived from the above Markov matrix P, under which $Y(t)$ can be represented by a process which can be described as the

[7] This Markov matrix P is central to the derivation of the representation of the process $Y(t)$ as a diffusion process.

[8] The notion of evolutionary entropy used in this paper is to be contrasted with the notion of entropy used in thermodynamics. Boltzmann and Gibbs defined a statistical notion of entropy $S = -\Sigma_j p_j \log p_j$, where p_j is the probability that a randomly chosen particle is in energy state j, while Clausius characterized it in energetic terms, $dS = \frac{dQ}{T}$, where Q is the heat of the material body and T is its temperature. The theormodynamic notion of entropy applies to isolated systems and hence to inert matter. It also measures the extent of energy spreading and sharing, and is thus generalized by the notion of evolutionary entropy which applies to living organisms and biological systems. The evolutionary entropy notion also has an energetic characterization besides the statistical one given in this paper. For more discussion, see Demetrius [38].

[9] In the introduction we also, loosely, interpret the evolutionary entropy as measuring the degree of cooperation of the society. This link is studied explicitly in Demetrius and Gundlach [39] within a biological context. The idea is that a strong degree of interaction measured by the number of pathways between individuals in different classes reflects more mutualistic and less selfish behavior. A simple example is sketched in Appendix A.5.

[10] This is a result that is well studied in stochastic population dynamics following the branching processes originally analysed by Feller [44] (see Dawson [36] and Haccou et al. [53] for more recent expositions). As mentioned above, in these models there is a natural unit, an individual in the population; in the present approach, we assume the existence of a "representative" unit transacted that is also indivisible. Our source of fluctuations is apparently similar, but is distinct and complementary, to the ones of Acemoglu et al. [2] and Gabaix [47], who model aggregate fluctuations through the propagation of individual firm-level shocks across the economy as a function of the network structure and the relative sizes of the firms.

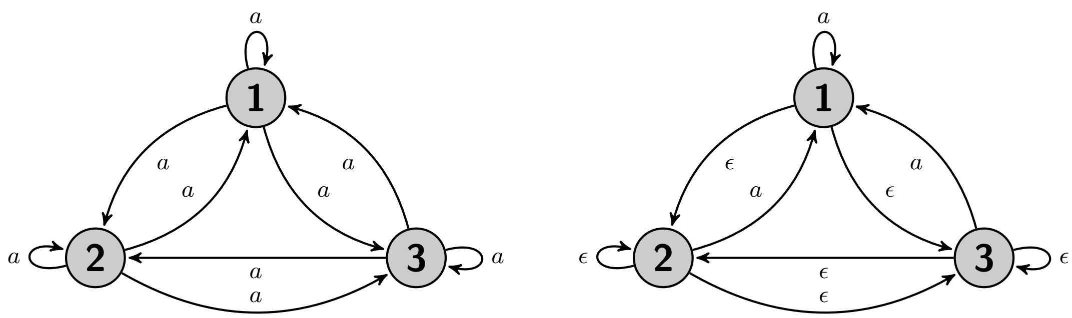

Figure 1: High entropy (left) and low entropy (right) interaction graphs, $a \gg \epsilon > 0$, $\epsilon \approx 0$.

solution to the following continuous time stochastic differential equation,

$$dY(t) \quad = \quad rY(t)dt + \sigma\sqrt{Y(t)}dW(t), \tag{7}$$

where $W(t)$ is Brownian motion, and $r \in \mathbb{R}$ and $\sigma\sqrt{Y(t)} \geq 0$ describe respectively the growth rate and the standard deviation of the process; the macroscopic variable σ is defined in the next subsection.[11] Essentially, the process $Y(t)$ can be viewed as a branching process, where in any given interval of time different "representative" units of income are generated and transacted according to rates derived from the matrix A. This gives rise to the Feller-type process described in (7) that ultimately only depends on two parameters, r and σ^2 (see Dawson [36] and Haccou et al. [53]; and also Demetrius et al. [41]). It belongs to the class of Cox-Ingersoll-Ross (CIR) processes also studied in finance (see Cox et al [34]).

Macroscopic Variables. The evolution of aggregate income is subject to variations, perturbations, and shocks, that can originate in a variety of ways, reflecting different types of innovations, technological, organizational, political, or cultural, but also events such as small-scale conflicts, epidemics, immigration among many others. Indirectly the interaction matrix A embodies the degree of efficiency in appropriating, transforming, and allocating resources, commodities, or income. In many cases these variations (perturbations, innovations, or shocks) affect the production process and the way the society interacts. In our analysis they are captured by small changes in the interaction matrix. It is important therefore to characterize how the evolution of aggregate income responds to such small perturbations of the interaction matrix. In order to keep things tractable while attempting to capture a large class of such variations, we consider, throughout the paper, perturbations of the form,

$$A(\delta) = (a_{ij}^{1+\delta})_{ij} \equiv A^{1+\delta}, \text{ for } \delta \in \mathbb{R}. \tag{8}$$

These constitute in a precise sense a canonical class of perturbations that can be modeled with a one-

[11] This process is likely to underestimate the actual variance, see Stock [79] who estimates a variance of the order $Y(t)$ rather than $\sqrt{Y(t)}$ for postwar US GNP. To better understand the difference, write the discrete time version of our process as $Y(t) = (1 + r)Y(t-1) + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma^2 Y(t))$, and notice that our process yields an error term that decreases in $Y(t)$, hence vanishes in the limit, when estimated in log-differences ($\Delta \log Y(t)$) rather than a stationary one as in [79]. On the other hand, the present model seems to explain a nontrivial amount of fluctuations. These fluctuations vanish if N or Y has the cardinality of the continuum as is confirmed in corresponding macro-models, where stationary idiosyncratic shocks typically do not lead to aggregate fluctuations (e.g., Aiyagari [8], Bewley [21]). In other words, the size of the "representative" unit of income transacted matters in determining the magnitude of fluctuations explained.

dimensional parameter, $\delta \in \mathbb{R}$.[12] These perturbations play a fundamental role in our analysis (comparable to mutations in biological models). Importantly, they generate a number of *macroscopic parameters* that measure diverse aspects of the income process $Y(t)$ besides its growth rate which has been almost exclusively at the center of most economic growth theory (e.g., Acemoglu [1] and Galor [49]). In our analysis, the growth rate r and the evolutionary entropy H are the two most important ones, but as we will see there are several others that also play a vital role.

We capture the effect of a perturbation on the steady state growth path by means of a Taylor expansion of the growth rate of aggregate production,

$$r(\delta) = r(0) + \delta r'(0) + \frac{\delta^2}{2!} r''(0) + \frac{\delta^3}{3!} r'''(0) + \cdots, \tag{9}$$

where $r(\delta) = \log \lambda(\delta)$ is the perturbed growth rate. From this we derive various **macroscopic parameters** that are useful in characterizing the long-run evolution of the process $Y(t)$. As we will see, besides the growth rate r and the evolutionary entropy H, a fundamental parameter is the first derivative of the growth rate, $\Phi = r'(0)$; other relevant parameters are the second and third derivatives, $\sigma^2 = r''(0)$ and $\kappa = r'''(0)$ respectively; another parameter that plays an important role is $\gamma = \frac{\partial \sigma^2}{\partial \delta}\Big|_{\delta=0}$, which is the first term of the Taylor expansion of $\sigma^2(\delta)$.[13] An important feature of the present evolutionary approach is that the signs and magnitudes of the first three moments matter in the long-run evolution of aggregate production and its distribution. Consistent with the literature, we sometimes refer to Φ and γ as respectively the *reproductive potential* and the *demographic index*; as we will see, they are the macroscopic variables representing, respectively, constancy and diversity of the income process (see the next subsection below), and they can be shown to satisfy the relations $\Phi = r - H$ and $\gamma = 2\sigma^2 + \kappa$ (see Appendix A.3 for a derivation of this and other relations between the macroscopic variables).

Constancy and Heterogeneity of the Economy. The entropic selection principle, as the central tenet of our theory, distinguishes the income generation processes or the productive environment in which the economy operates along two dimensions. On one hand it distinguishes environments by how steadily the income process evolves, and on the other, by how rich and diverse their base of income generation is. We refer to these two aspects as the **constancy** and **heterogeneity** dimensions. As is clear especially from the proof of the entropic selection principle, these two aspects are determinant for the direction of the society's cultural-evolutionary behavior. We define them formally.

Constancy: We say the income generation process is **constant** or steady if $\Phi < 0$, that is when the growth rate r is smaller than the evolutionary entropy H, so that the coefficient $\Phi = r - H$ is negative; we say it is **fluctuating** or unsteady if $\Phi > 0$, that is, when the growth rate is larger than the entropy, so that $\Phi = r - H$ is positive.

As will be clear from the Perturbation Stability Theorem (Theorem 4) the evolutionary entropy H is a direct measure of the rate at which the economy returns to steady state after any given shock. This suggests that when $\Phi = r - H > 0$ the economy grows faster than it returns to steady state, thus resulting in an unsteady or what we call "fluctuating" environment; when $\Phi < 0$ the economy grows at a slower

[12] We refer to Arnold et al. [14], Demetrius et al. [41], and also Demetrius [38], Ch. 4, for a more formal statement of this fact; we provide a sketch of the argument in Appendix A.3. Perturbing individual entries of the matrix A can be both arbitrary and may violate certain feasibility conditions of how resources can be transformed and transferred across the society given its level of techology. The one-parameter family we consider is to be seen as a tractable compromise to capture a wide class of phenomena.

[13] By this we mean the Taylor expansion associated to $\sigma^2(\delta') = r''(0)(\delta')$ when further perturbing the perturbed matrix $A(\delta)$ again by δ', giving the matrix $(A(\delta))(\delta')$.

rate than the one with which it returns to steady state, thus resulting in a more steady or what we call "constant" environment. In Appendix A.4 we also formally show a positive correlation between Φ and the standard economics measure of volatility of the income process $Y(t)$, namely, the variance of the short term growth rate of aggregate income, by showing that it is typically positively correlated with the measure of instantaneous volatility σ^2.

Heterogeneity: We say the income generation process is **diverse** if $\gamma > 0$, that is when the kurtosis measure κ is nonnegative or when the variance measure σ^2 is large relative to the kurtosis measure κ, so that the coefficient $\gamma = 2\sigma^2 - \kappa$ is positive; we say it is **singular** if $\gamma < 0$; this can only occur if the kurtosis measure κ is negative and the variance measure σ^2 relatively small.

Given the nonnegativity of the variance, a small or negative value of γ essentially reflects a negatively skewed income process. This is typical of economies with extremely few but very important activities rather than many relatively not so dominating ones.[14]

Population. As we will discuss in more detail below, we also assume that total population $N(t)$ follows a stochastic process described by the solution to the stochastic differential equation

$$dN(t) = \widetilde{r}N(t)dt + \widetilde{\sigma}\sqrt{N(t)}dW(t), \tag{10}$$

where again $W(t)$ is Brownian motion in common with the income process $Y(t)$. Thus to capture the fact that individual reproduction is positively related to the income of the individuals, we assume that in the steady state the population process $N(t)$ is coupled with the production process $Y(t)$ by means of a linear relation (F, f) such that $N(t) = F(Y(t))$ and $n(t) = f(y(t))$, where $F', f' > 0$. It is important to point out that while we assume the processes $Y(t)$ and $N(t)$ are positively and linearly related, what really matters for our results is not so much the linearity but rather a strictly positive relation between the variables that has second order derivatives bounded from below. This ensures that key macroscopic variables such as the growth rates and variances of the two processes are strictly positively correlated (as specified in Appendix A.4). We will describe the population process in more detail in Section 4 below. We assume linearity of the relation (F, f) for simplicity and in order to be able to characterize the processes $Y(t)$ and $N(t)$ as diffusions satisfying respectively (7) and (10) with common process $W(t)$.

Cultural Evolution and Entropic Selection Principle. Having described the main ingredients of the society's environment and income generating process, we can now turn to the evolutionary aspects of inequality and mobility. We use a model of **cultural evolution** that is based on the interaction between an incumbent population and a variant (or mutant) sub-population that can potentially increase in frequency and lead to a displacement of the traits of the original types. This can be seen as being representative of various models of cultural evolution. We calculate the probability that the traits of the variant population (in our case these are derived from their respective interaction matrix) take over the whole population and relate this event to macroscopic parameters of the underlying income processes of the incumbent and variant populations.

[14] Our framework does not directly distinguish different "types" of activities. As such we cannot give a formal derivation of the connection between our measure γ and an empirically observable notion of diversity. However, as is clear from Appendix A.4, the variable σ^2 is a measure of variance derived from the a_{ij} terms. A large number of activities will likely be associated with a large such variance and will result in a positive value of $\gamma = 2\sigma^2 + \kappa$. On the other hand, a small number of activities is likely to be associated with a small variance and, moreover, if the productivities associated are also concentrated in few highly performing activities, this is likely to generate a distribution of the a_{ij}'s which is negatively skewed (high density on few activities with high values of a_{ij}), this will lead to a negative kurtosis κ, and can result in a negative value of γ. We leave a formalization of this relationship for future research.

More specifically, to the incumbent population N operating with an interaction matrix A as described above and producing Y, we consider a (small) variant population N^* operating with an interaction matrix A^* that is modeled as a perturbation of the one of the incumbent population, $A^* = A(\delta)$, and which produces Y^*. The population N^* can be thought of as a sub-population like a community, city, or county that operates in a manner similar to A but slightly different. Concretely we set $A^* = A(\delta)$, for any given δ. This perturbed matrix A^* is meant to represent a slightly different modes of resource exchange or exploitation, that can originate from technological innovation, cultural, ethnic, or political differences among others. The question is then whether this variant will increase in frequency and displace the taits of the original population, or simply disappear, which is calculated based on the interaction of the implied processes $Y(t)$ and $Y^*(t)$.

The introduction of a variant type thus constitutes the first event in describing the **evolutionary dynamic**. The second event is the competition between the incumbent population N and the variant population N^* for the resources which the economic environment produces for their corresponding productions Y and Y^*. The evolutionary entropies H and H^* associated with A and A^* respectively are indices summarizing the interaction modes and ultimately the social preferences of the populations N and N^*. Our model naturally assumes that resources are limited in abundance. This means that the outcome of the competitive process between the two populations will be decided by the respective rates at which they can appropriate resources and generate income from their respective environments. As we will see, the selective outcome is then decided by the evolutionary entropies H and H^* of the two populations and is also contingent on the variability and heterogeneity of the environment.

Formally, the (global) selective dynamic is determined by the **entropic selection principle**. As studied in directionality theory, the dynamical changes in evolutionary entropy under the process of variation and selection are contingent on the resource abundance and can be characterized in terms of the following local result:

(I) When the income generating process is *constant* and *diverse*, a community with higher entropy will have a selective advantage and increase in frequency.

(II) When the income generating process is *fluctuating* and *singular*, a community with low entropy will have a selective advantage and will increase in frequency.

We will exploit *(I)* and *(II)* to resolve a series of problems on the origin, spread, and persistence of inequality, which have hitherto seemed intractable within classical frameworks of economic growth and cultural evolution. The entropic selection principle can also be interpreted loosely as characterizing when cooperative vs. self-serving behavior, or more vs. less egalitarian social preferences, will spread locally.

3 Two Examples

To illustrate our basic evolutionary framework, we embed it in two different but otherwise standard economic models of growth. Both models allow for two or more classes or sectors in the economy, whose aggregate income or production shares all grow at the same constant rate in steady state. These examples allow us to introduce our theory within familiar economic frameworks, where agents work, produce, consume and save, and to interpret and derive from basic preference and technology parameters the interaction matrix A that is at the center of our analysis within such contexts. The first example, which is an adaptation of the basic model of Solow and Samuelson [75] considers a multi-sector economy, in which the classes correspond to production sectors. We derive and interpret the law of motion for such an economy. The second example builds on a one-sector model of Alesina and Rodrik [11]. In our adaptation individuals from two different classes work together to produce a single homogeneous good that is used for both consumption and capital. Again, we derive the law of motion for a simple calibration of this model. None of these models studies the stochasticity of the income process nor uses it to derive an evolutionary analysis. That is what our theory adds to these otherwise classic models.

Example 1. Following Solow and Samuelson's [75] model mentioned in footnote 4 above, consider a closed economy with d sectors such that each sector produces its commodities using as potential inputs, commodities produced by sectors $1, \ldots, d$. Suppose also that individuals are associated to a sector in the sense that they work in just one sector. The population is then evenly distributed across d classes that coincide with the d sectors. Suppose also that individuals in the different classes consume (as outputs) commodities from the same sectors $1, \ldots, d$. Without modeling the details of the individual consumption and production decisions, assume the relative proportions are all linearly additive.[15] Then, let $y(t) = (y_1(t), \ldots, y_d(t))$, where $y_i(t)$ is total income of class i in period t. We can write the law of motion for the corresponding economy in the form the following difference equations,

$$y_i(t+1) = a_{i1}y_1(t) + \ldots + a_{id}y_d(t), \text{ for } i = 1, \ldots, d,$$

or more compactly as,

$$y(t+1) = Ay(t),$$

where $A = (a_{ij})$ is an interaction matrix. Since there is a consumption and a production dimension in this economy, the elements of the interaction matrix A embody both a production and a consumption component; transactions due to both consumption and production affect the total rate a_{ij}, which is used to describe the society's intertemporal income generation process. Recall that a_{ij} represents the contribution of one unit of income in class j in period t to the income of class i in period $t+1$.

From the matrix A one can derive all the macroscopic variables necessary for the analysis of the paper, such as the eigenvalue λ, the growth rate r, the entropy level H and so on. By looking at Φ and γ one can determine whether the income process is fluctuating or steady and whether it is singular or diverse. By invoking the theorems to follow, this allows us to deduce whether the economy has a tendency towards a higher or lower level of entropy and similarly for measures of inequality and social mobility. The next example allows agents from different classes to work in the same sector.

Example 2. Consider the following model, inspired from Alesina and Rodrik [11], of a one-sector economy with production function,

$$Y(t) = K(t)^{\alpha}G(t)^{1-\alpha}L(t)^{1-\alpha}, \ 0 < \alpha < 1,$$

and multiple classes of agents (indexed by relative factor endowments) which we collapse into two extremes, namely, those endowed with only capital (class 1) and those endowed with only labor (class 2), and where

[15] This can be made consistent with a simplified multi-sector growth model along the lines of, for example, Acemoglu [1], Ch. 20. However, while the present example provides a clear interpretation of the matrix A, the next example with one sector and two classes seems more representative of modern industrialized economies, where individuals from different classes can work in the same sector. It would be interesting to embed our evolutionary approach within a unified growth theory model such as Galor and Moav [50], see also Galor [48, 49]; we leave this for future work.

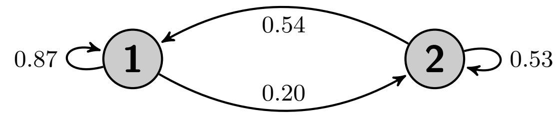

Figure 2: Interaction graph for economy with parameters $\alpha = 0.6, \rho = 0.04$, and $\tau = 0.02$.

all agents have the same utility function,

$$U = \int \log c(t)\, e^{-\rho t} dt, \;\; \rho > 0.$$

Moreoever, there is a government sector which we subsume to the worker class, and which is funded through a capital tax: $G(t) = \tau K(t)$; set $L(t) = 1$ for all t. The income shares from the production activity can be written as $\widetilde{\alpha} = \alpha - \tau^{\alpha}$ for class 1, and $1 - \widetilde{\alpha} = 1 - \alpha + \tau^{\alpha}$ for class 2, so that the actual income, consumption, and savings are given by $y_1(t) = (\alpha\tau^{1-\alpha} - \tau)K(t)$, $c_1(t) = \rho K(t)$, $s_1(t) = y_1(t) - c_1(t)$, for class 1 and by $y_2(t) = ((1-\alpha)\tau^{1-\alpha} + \tau)K(t)$, $c_2(t) = y_2(t)$, $s_2(t) = 0$ for class 2.[16]

Notice that the agents in the economy jointly participate in the production of the (single) commodity and are therefore naturally linked through the processes of consumption and production. From the point of view of our framework, this means that we can write down an interaction matrix for this exchange economy that takes these flows into account. We therefore extend the original model of Alesina and Rodrik, which does not have any interaction matrix, and derive, from the basic parameters of the model, such a matrix A for the exchange economy described.

Let $\widetilde{c}_i = \frac{c_i(t)}{y_i(t)}$ be the proportion of income spent on consumption by individuals in class i, for $i = 1, 2$. Income spent on consumption is divided to the two classes according to the shares of wages and capital income. Therefore, consumption expenditures by agents in class 1, ultimately constitute flows of income from class 1 to both class 1 and class 2; and similarly for class 2. We also record savings (only relevant for class 1) as flows from class 1 to itself. Multiplying these static flows by $1 + r$ to allow for growth yields the entries for the interaction matrix A:

$$a_{11} = (\widetilde{c}_1\widetilde{\alpha} + (1 - \widetilde{c}_1))(1 + r),\; a_{12} = \widetilde{c}_2\widetilde{\alpha}(1 + r),\; a_{21} = \widetilde{c}_1(1 - \widetilde{\alpha})(1 + r),\; a_{22} = \widetilde{c}_2(1 - \widetilde{\alpha})(1 + r),$$

where $\widetilde{c}_1 = \frac{\rho}{\alpha\tau^{1-\alpha} - \tau}$, $\widetilde{c}_2 = 1$. Note also that, $v = (v_1, v_2) = \left(\frac{\widetilde{\alpha}}{(1-\widetilde{\alpha})\widetilde{c}_1}, 1\right)$.

To give a more concrete idea, we can calculate the matrix A and corresponding macroscopic parameters for the values $\alpha = 0.6, \rho = 0.4$, and $\tau = 0.02$, and obtain:

$$A = \begin{pmatrix} 0.87 & 0.54 \\ 0.20 & 0.53 \end{pmatrix},\; P = \begin{pmatrix} 0.81 & 0.19 \\ 0.51 & 0.49 \end{pmatrix},$$

with eigenvalue and (normalized) eigenvector (for A),

$$\lambda = 1.07,\; v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 2.7 \\ 1 \end{pmatrix},$$

[16] More precisely, in the original model agents differ by their endowments in labor and capital; here we assume agents have either a fixed capital and zero labor endowment (class 1) or a fixed labor and zero capital endowment (class 2).

and macroscopic parameters,

$$r = 0.07,\ \sigma^2 = 0.35,\ H = 0.54,\ \Phi = -0.48,\ \gamma = 0.75.$$

From this, we deduce that the corresponding economy is constant and diverse, which in our framework means that, under a socio-cultural evolutionary process, the entropy will tend to increase and the income shares of the two classes will tend to increase their interactions. In our model, this can happen in many ways that are picked up by "perturbations" of the matrix A. Such perturbations that lead to higher entropy will survive and displace the original matrix in this case. This is a different yet complementary mechanism from the one of Alesina and Rodrik's original model, where taxation and hence the amount of redistribution is decided through voting and depends on the distribution of the relative factor endowment ratios across individuals in the population. We view our approach as adding a further constraint or source of redistributive pressure as a function of the basic parameters Φ and γ of the underlying steady state income generation process.

4 The Entropic Selection Theorem

In this section, we state and discuss the Entropic Selection Theorem (Theorem 1), which is central to our overall approach. It also constitutes a core result from directionality theory (Demetrius [38]). In our context, it relates the environment of the economy – its constancy and heterogeneity – to the evolution of the entropy measure H. In the next section we will further relate the entropy measure to economic measures of inequality and of social and income mobility. The following theorem therefore will have implications for redistribution within societies as well as its persistence. In a subsequent section, we also derive consequences concerning the robustness or resilience of societies.

Entropic Selection. The Entropic Selection Principle is a *local* result. It pertains to the outcome of competition locally between an original or ancestral population defined in terms of its mode of allocating resources, and a variant population defined in terms of a related mode. In any natural population and production process new variants are continually being introduced. At each instant the variant types must compete with the original types for the existing resources. A variant type will persist if it has a selective advantage or it will disappear if the original type has the selective advantage.

Consider a population with an income process evolving as in the framework of Section 2. The next theorem identifies a central variable, the evolutionary entropy H, as characterizing what types of interactions evolve and concomitantly what kind of social dispositions will proliferate. The question we ask here concerns the long-term change in evolutionary entropy as new variants arise.

Our analysis shows that whether higher or lower entropy interactions prevail, depends on characteristics of the underlying economic environment. Steady and diverse environments are conducive towards higher entropy interactions, while unsteady and singular ones are conducive towards lower entropy interactions. The following result states how the level of entropy of the underlying interaction will evolve *globally* in all possible cases.

Theorem 1 (Entropic Selection Theorem)**.** *The outcome of the selection process in a society evolving according to the income process $Y(t)$ described by Eq. (4) above is characterized by the following four cases:*

(Ia) If the income process is constant and diverse ($\Phi < 0, \gamma > 0$), entropy tends to increase;

(Ib) If the income process is constant and singular ($\Phi < 0, \gamma < 0$), entropy tends to increase, provided total income is sufficiently large ($Y > \gamma/\Phi$); otherwise for small total income ($Y < \gamma/\Phi$) entropy increases with a probability that increases in the total level of income;

(IIa) If the income process is fluctuating and singular ($\Phi > 0, \gamma < 0$), entropy tends to decrease;

(IIb) If the income process is fluctuating and diverse ($\Phi > 0, \gamma > 0$), entropy tends to decrease, provided that total income is sufficiently large ($Y > \gamma/\Phi$); otherwise for small total income ($Y < \gamma/\Phi$) entropy decreases with a probability that increases in the total level of income.

This suggests that it is essentially in constant environments ($\Phi < 0$) that one should expect to find higher entropy societies. In large economies the result is general. When aggregate income is not sufficiently large ($Y < \gamma/\Phi$) then, to guarantee the same result, one needs that the environment also be diverse ($\gamma > 0$). Conversely, it is when the environment is fluctuating ($\Phi > 0$) that one should expect to find low entropy societies. Again, in large economies this is general. When aggregate income is not sufficiently large ($Y < \gamma/\Phi$) then, to guarantee the same result, one needs that the environment also be singular ($\gamma < 0$).

As we will see in the next section, when we relate the evolutionary entropy measure H to a measure of income inequality (the Theil index T), this characterizes when to expect equal and when to expect unequal societies to prevail. It also suggests that when the economy is not sufficiently large, the outcome may be uncertain depending on the sign of the second-order variable (γ), which reflects the degree of heterogeneity of the income process. Notice that the model of cultural evolution implicit in our analysis is rather general and assumes neither an infinite population nor an infinite level of aggregate income.

The analytical basis for the Entropic Selection Theorem involves the integration of the ergodic theory of dynamical systems with the theory of diffusion processes (Demetrius [38]). The proof of the theorem is sketched below with some key steps proved in Appendix B.1. Before that, we offer the following intuition for the result.

Heuristics for the Entropic Selection Theorem. The basic intuition is that a society with high evolutionary entropy H is more stable, more mutualistic and better suited for a constant, diverse environment. By intensifying its link structure, it can adapt itself better and extract more out of such an environment; at the same time it does not have the appropriate flexibility to adapt to a fluctuating, singular environment. A society with low evolutionary entropy on the other hand is less stable, less mutualistic, and, having a relatively more sparse link structure, it can adapt more easily to changes and thus extract relatively more from a fluctuating, singular environment; at the same time it does not have the richness of pathways to do well in a constant, diverse environment. The mechanism operates at the individual level with individuals in different classes of a community or subpopulation interacting with individuals in the same and in other classes. Given a population with interaction structure A, a successful variant interaction structure A^* adopted by a subopulation can spread to the whole society. The entropic selection result suggests that in constant and diverse environments, societies consisting of individuals with more cooperative and mutualistic traits will tend to prosper, whereas in fluctuating and singular environments individuals with more selfish traits will tend to prosper.[17]

We now sketch the argument more formally. Besides the first-order effect of the constancy of the environment, it also includes the effect of heterogeneity. Further details of the proof are in Appendix B.1.

[17] Demetrius and Gundlach [39] studies more in detail the evolution of cooperation within a biological context, using the directionality approach. In Appendix A.5, we provide some brief discussion and sketch a connection with the notion of evolutionary entropy.

Invasion Dynamics. The essence of our argument lies in the interaction between the aggregate production of a given population (the incumbent population, described by A) and that of the variant population (described by A^*). We model variants (mutants or invaders) as having their own production and redistribution technology represented by a matrix A^*, which we model as being a perturbation $A(\delta) = A^{1+\delta}$ of the original matrix A, along with corresponding macroscopic parameters $r^* = r(\delta)$, ${\sigma^*}^2 = \sigma^2(\delta)$, $H^* = H(\delta)$, for $\delta \in \mathbb{R}$ small in absolute value. Their aggregate income follows a stochastic process described by,

$$dY^*(t) = r^* Y^*(t)dt + \sigma^* \sqrt{Y^*(t)} dW^*(t),$$

where $W^*(t)$ is a Brownian motion independent of $W(t)$. Therefore, if the total production is given by

$$Z(t) = Y(t) + Y^*(t)\,,$$

then, setting $\Delta r = r^* - r$, $\Delta\sigma^2 = {\sigma^*}^2 - \sigma^2$, and defining the **income share of the invaders** as,

$$p(t) = \frac{Y^*(t)}{Z(t)},$$

one can look at the stochastic processes defined by

$$dZ(t) = (r + p(t)\Delta r)\, Z(t)dt + \sigma\sqrt{(1-p(t))Z(t)}dW(t) + \sigma^* \sqrt{p(t)Z(t)}dW^*(t),$$

and

$$dp(t) = p(t)(1-p(t)) \left(\Delta r - \frac{\Delta\sigma^2}{Z(t)}\right) dt - \sigma p(t)\sqrt{\frac{(1-p(t))}{Z(t)}}dW(t) + \sigma^*(1-p(t))\sqrt{\frac{p(t)}{Z(t)}}dW^*(t),$$

and solve for the process $p(t)$. Assuming that total aggregate production is constant, $Z(t) = Y$, then the process $p(t)$ can be shown to be a diffusion process with drift:

$$\alpha(p(t)) = p(t)\,(1-p(t)) \left(\Delta r - \frac{\Delta\sigma^2}{Y}\right)$$

and variance:

$$\beta(p(t)) = \frac{p(t)(1-p(t))}{Y} \left(\sigma^2 p(t) + {\sigma^*}^2(1-p(t))\right),$$

and with a well-specified density. Notice that the process $p(t)$ depends in an important way also on the variances of the respective processes $Y(t)$ and $Y^*(t)$.

Letting $p_0 = p(0)$ denote the initial frequency of the mutant and $\rho(p_0)$ the probability that the diffusion process leads to an absorption in the state $p = 1$ (extinction of the incumbent population), one shows that,

$$\rho(p_0) = \frac{1 - \left(1 - \frac{\Delta\sigma^2}{{\sigma^*}^2} p_0\right)^{\frac{2Ys}{\Delta\sigma^2}+1}}{1 - \left(1 - \frac{\Delta\sigma^2}{{\sigma^*}^2}\right)^{\frac{2Ys}{\Delta\sigma^2}+1}}\,, \tag{11}$$

where $s = \Delta r - \frac{\Delta\sigma^2}{Y}$. The sign of the expression s is then crucial in determining whether a variant is successful in invading or not. The process $p(t)$ is absorbed in 0 (invaders disappear) for any possilble perturbation, if $\Delta r < 0$, $\Delta\sigma^2 \geq 0$, or $\Delta r \leq 0$, $\Delta\sigma^2 > 0$; that is, when $s < 0$ where we have $\Delta r < \frac{\Delta\sigma^2}{Y}$

meaning that the incumbent's growth rate advantage is greater than its normalized variance disadvantage; such an incumbent will prevail. On the other hand, the process $p(t)$ is absorbed in 1 (invaders take over) for any possilble perturbation, if $\Delta r > 0$, $\Delta\sigma^2 \leq 0$, or $\Delta r \geq 0$, $\Delta\sigma^2 < 0$, that is, when $s > 0$ where we have $\Delta r > \frac{\Delta\sigma^2}{Y}$ meaning that the invader's growth rate advantage is greater than its normalized variance disadvantage; in which case the invader will prevail.

From the definitions of the macroscopic parameters (see Appendix A.4), if $\Phi \neq 0, \gamma \neq 0$, and $\sigma^2 \neq 0$, we have the relations, $\Delta r \approx \Phi\delta, \Delta\sigma^2 \approx \gamma\delta, \Delta H \approx -\sigma^2\delta$, where $\delta \in \mathbb{R}$, $\delta \approx 0$, is the perturbation parameter. In order to express the selective advantage for the limiting case where $\delta \to 0$, we use the more general formula,

$$s = -\left(\Phi - \frac{\gamma}{Y}\right)\Delta H, \tag{12}$$

where $\Delta H = H^* - H$. This now describes the selective advantage directly in terms of the macroscopic parameters of the steady state income process $(Y(t))$ and of the entropy differential between the two populations (ΔH). This covers a very wide class of models of cultural evolution.[18]

5 Evolutionary Entropy, Inequality, and Social Mobility

In this section, we establish the implications of the Entropic Selection Theorem for the evolution of inequality and social mobility of societies. In order to state our main results, Theorems 2 and 3, we first introduce measures of income inequality and social and income mobility which we relate formally to corresponding measures of evolutionary entropy.

Population Evolution and Social Mobility. So far we have introduced the social classes and the stochastic process representation of the aggregate population (10) and have not directly used the evolution of the population. To address issues of mobility and inequality, we discuss the process of population evolution in more detail. As with the processes of aggregate production $(Y(t))$ and resources $(X(t))$, we consider the population process, $N(t) = \sum_{i=1}^{d} n_i(t)$, in steady state

$$n(t+1) = Bn(t), \tag{13}$$

with associated **population matrix**,

$$B = (b_{ij}), \qquad b_{ij} \geq 0, 1 \leq i, j \leq d, \tag{14}$$

[18] An important aspect of the present approach is that it assumes neither an infinite (population or) level of aggregate production ($Y \to \infty$) nor an infinite amount of available resources ($X \to \infty$). Instead the latter two are special cases of the present, more general, approach. The following diagram summarizes the selective advantages corresponding to the different cases.

$$\begin{tikzcd}
& s = -\left(\Phi - \frac{\gamma}{Y}\right)\Delta H \arrow[dl, "Y \to \infty"'] \arrow[dr, "X \to \infty"] & \\
s = -\Phi\Delta H \arrow[dr, "X \to \infty"'] & & s = \Delta r - \frac{\Delta\sigma^2}{Y} \arrow[dl, "Y \to \infty"] \\
& s = \Delta r &
\end{tikzcd}$$

The Malthusian case where the selective advandage s is based exclusively on the growth rate differential (Δr) between the two populations, is obtained in the limiting case where resources and total production (or population) are infinite (see Demetrius [38], Sections 2.3 and 6.3, for further discussion).

where b_{ij} measures the rate at which individuals from class j contribute to individuals of class i, which is to be interpreted as representing rates of how individuals transfer from one class to another. To simplify the analysis we maintain the assumption that the classes while having possibly different amounts of income, have the same population size, $n_i(t) = N(t)/d$; this amounts to assuming that the rows have constant sum. Again, we are interested in the steady state distribution of the population across classes, where here the right eigenvector takes the form $w = (w_1, \ldots, w_d) = \widetilde{\lambda}(1, \ldots, 1) \in \mathbb{R}^d_+$, where $\widetilde{\lambda} \in \mathbb{R}_+$, is the dominant eigenvalue such that $Bw = \widetilde{\lambda} w$. The steady state share of population that is in class i, $i = 1, \ldots, d$, is always given by $\overline{w}_i = 1/d$. As above, $\widetilde{r} = \log \widetilde{\lambda}$ denotes the steady state growth rate of the population.

From this we can also define the Markov matrix which here takes the simpler form

$$Q = (q_{ij}) = \left(\frac{b_{ij} w_j}{\widetilde{\lambda} w_i} \right) = \widetilde{\lambda}^{-1} B, \qquad 0 \leq q_{ij} \leq 1, 1 \leq i, j \leq d.$$

It describes the probabilities with which individuals move across classes. An element $q_{ij} \geq 0$, can be interpreted as the probability that, in the steady state, an individual in class i came from class j. Defining $\rho = (\rho_1, \ldots, \rho_d)$ as the corresponding stationary distribution satisfying $\rho Q = \rho$, we can define the entropy of the population

$$\widetilde{H} = -\sum_{i=1}^{d} \rho_i \sum_{j=1}^{d} q_{ij} \log q_{ij}, \quad 0 \leq \widetilde{H} \leq \log d. \tag{15}$$

To the extent that the d classes are social classes, or describe occupational choices, this measure of entropy is a natural measure of **social mobility** in that it represents precisely the extent and intensity of flows between the d classes. We can again derive the other macroscopic variables $\widetilde{r}, \widetilde{\Phi}, \widetilde{\sigma}^2, \widetilde{\gamma}, \widetilde{\kappa}$, which satisfy the analogous relations as the corresponding variables for A. Moreover, the macroscopic variables associated to the population process are also positively correlated with the corresponding variables derived for the matrix A. We come back to the evolution of social mobility in the next section.

Theil Index and Evolution of Income Inequality. We here establish a connection between certain measures of entropy and the Theil index, a well-known entropy-based measure for the income distribution of an economy, that allows to decompose overall inequality into across-group and within-group inequality (see, e.g., Cowell [33]). In our case, we only conisder across group inequality and we do not normalize the index to lie between $[0, 1]$. Theil's index can then be written as,

$$T(v) = \frac{1}{d} \sum_{i=1}^{d} \frac{v_i}{\bar{v}} \log\left(\frac{v_i}{\bar{v}}\right), \quad 0 \leq T(v) \leq \log d, \tag{16}$$

where $\bar{v} = \sum_{i=1}^{d} v_i/d$ is the steady state average income of a class i; and v is the income distribution corresponding to the eigenvector of A; we also write this as $T(0) \equiv T(v)$, and write $T(\delta) \equiv T(v(\delta))$ for the Theil index of the distribution of the eigenvector of the perturbed matrix $A(\delta)$.

Proposition 1 (Evolutionary Entropy and Inequality)**.** *For perturbations of the form $A^{1+\delta}$, we have that the Theil index and the evolutionary entropy move in opposite directions, $\Delta T \Delta H < 0$, where $\Delta T = T(\delta) - T(0)$ and $\Delta H = H(\delta) - H(0), \delta \in \mathbb{R}$ small.*

This result is crucial in linking inequality as measured by T with the evolutionary entropy H.

Furthermore, we can define the following notion of income mobility (see Fields [45, 46]) that compares two (successive) income distributions, say $v = (v_1, \ldots, v_d)$ and $v' = (v'_1, \ldots, v'_d)$, defined by

$$E(v, v') = 1 - \frac{T(\frac{v+v'}{2})}{T(v)}. \tag{17}$$

As is discussed in Fields [45, 46], this is a measure of the degree of equalization of income distributions v and v', in the sense that positive values indicate a higher income mobility at v' since average income $\frac{v+v'}{2}$ is more equally redistributed than base income v with respect to the inequality measure T. Of course, income mobility can be defined with other inequality measures besides T, such as the Gini coefficient. We also write $E(\delta, 0) \equiv E(v(\delta), v(0))$.

Redistributive Selection. We now turn to our main implication, which establishes a link between the nature of the economic environment and the evolution of inequality in the society. Given the last proposition, the notion of evolutionary entropy plays a central role in establishing the link. Again, in our evolutionary model, variants with different modes of redistribution (perturbed matrices A^* with levels of inequality T^*) are continually introduced; these variants have to compete with the original types (operating with matrix A with level of inequalty T) for the existing resources. The outcome of the evolutionary process depends on which of the two types has a selective advantage. This in turn depends on the nature of the economic environment.

The following result contains our characterization of the evolution of inequality (measured by T).

Theorem 2 (Redistributive Selection Theorem)**.** *The outcome of the selection process facing a society evolving according to processes $Y(t)$ and $N(t)$ described by respectively Eqs. (4) and (13) above is characterized by the following four cases:*

(Ia) If the income process is constant and diverse ($\Phi < 0, \gamma > 0$), income inequality tends to decrease;

(Ib) If the income process is constant and singular ($\Phi < 0, \gamma < 0$), income inequality tends to decrease, provided total production is sufficiently large ($Y > \gamma/\Phi$); otherwise for small total production ($Y < \gamma/\Phi$), income inequality tends to decrease with a probability that increases in total production;

(IIa) If the income process is fluctuating and singular ($\Phi > 0, \gamma < 0$), income inequality tends to increase;

(IIb) If the income process is fluctuating and diverse ($\Phi > 0, \gamma > 0$), income inequality tends to increase, provided total production is sufficiently large ($Y > \gamma/\Phi$); otherwise, for small total production ($Y < \gamma/\Phi$), income inequality will tend to increase, with a probability that increases in total production.

This result follows as a direct application of the Entropic Selection Theorem (Theorem 1) and Proposition 1. It shows that, whether equal or unequal societies tend to prevail depends on the constancy and diversity of the environment. More equal societies tend to emerge in constant and diverse environments, while less equal ones tend to emerge in fluctuating and singular ones. Moreover, more equal societies also grow faster in constant and diverse environments (than less equal ones), and, conversely, less equal societies grow faster in fluctuating and singular environments (than more equal ones). Thus, whether more equal or less equal societies grow faster can also depend on the constancy and heterogeneity of the underlying environment.

Income and Social Mobility. Next we relate our measures of income inequality (T), social mobility ($\widetilde{H}$), and income mobility (E), introduced above to each other and to our central notion of evolutionary entropy (H); this readily yields a characterization of the selective advantage of societies, also in terms of income and social mobility for different environments.

Our measure of income mobility E defined using the Theil index can be shown to be positively correlated with our measure of entropy H.

Proposition 2 (Evolutionary Entropy and Income Mobility)**.** *For perturbations of the form $A^{1+\delta}$, we have that income mobility as measured by E and changes in the evolutionary entropy H are positively correlated, $E\Delta H > 0$, where $\Delta H = H(\delta) - H(0), \delta \in \mathbb{R}$ small.*

In other words, small perturbations that increase the entropy H will be associated with a positive income mobility E. Together with Proposition 1 this readily implies that changes in inequality are negatively correlated with income mobility, $E\Delta T < 0$, that is, along perturbations of the form $A^{1+\delta}$, changes in the Theil index and the income mobility measure are negatively correlated. This relationship has been recently called the "Great Gatsby Curve" by Alan Krueger (see Corak [31] and the 2012 US Report of the President [32]); we will come back to it in Section 7. It suggests that societies with higher levels of income inequality also tend to exhibit lower levels of (intergenerational) income mobility and vice versa. In our theory, the key link between these two measures is given by the notion of evolutionary entropy of the income process. An implication of this is that a highly stratified society will tend to be highly immobile and hence remain stratified.

Furthermore, given the positive (linear) relation (F, f) assumed between the income and the population process, it immediately follows that the entropy measure H and the social mobility measure $\widetilde{H}$ are also positively correlated.

Proposition 3 (Evolutionary Entropy and Social Mobility)**.** *For perturbations of the form $A^{1+\delta}$, we have that social mobility as measured by the entropy measure $\widetilde{H}$, and the evolutionary entropy H move in the same direction, $\Delta\widetilde{H}\Delta H > 0$, where $\Delta\widetilde{H} = \widetilde{H}(\delta) - \widetilde{H}(0)$ and $\Delta H = H(\delta) - H(0), \delta \in \mathbb{R}$ small.*

In other words, small perturbations that increase the entropy H will also increase social mobility as measured by $\widetilde{H}$. Again, together with Proposition 1 this readily implies that changes in inequality are negatively correlated with changes in social mobility, $\Delta T \Delta\widetilde{H} < 0$. Thus, along perturbations of the form $A^{1+\delta}$, the Theil index of inequality and the social mobility measure move in opposite directions.

Together with the entropic selection theorem we can then show the following result on mobility.

Theorem 3 (Income and Social Mobility Theorem)**.** *Societies with a constant and diverse income process ($\Phi < 0, \gamma > 0$) will tend towards higher levels of income and social mobility; ones with fluctuating and sigular income process ($\Phi < 0, \gamma > 0$) will tend towards less income and social mobility.*

In constant and diverse environments more mobile societies will tend to prevail, while in fluctuating and singular ones less mobile and more stratified societies will tend to prevail. Together with Theorem 2 this shows that steady and diverse environments are associated with both more equality and more income and social mobility, whereas fluctuating and singular ones are associated with less equality and less mobility. This provides a new explanation, based on the environment and the evolution of cooperation, for the "Great Gatsby curve," which finds a negative relation between mobility and inequality. The main result of the next section (Theorem 4) shows that more stratified societies are more vulnerable to shocks and thus more fragile than more egalitarian ones.

6 Robustness and Inequality

Do equal societies present advantages over unequal ones? Wilkinson and Pickett [83], drawing from evidence carried out by a large number of researchers also outside of economics, suggest that economic inequality is "socially corrosive" in that it is positively correlated with a variety of "undesirable" indices that cover issues ranging from health and mental disorders, life expectancy and literacy rates to fairness, trust, and happiness of individuals in society. In economics, a number of papers have studied the effect of inequality on a country's economic performance, specifically on its rate of growth, Aghion, Caroli, and García-Peñalosa [7], Banerjee and Duflo [18], Bénabou [19], Galor and Zeira [51], and Persson and Tabellini [66]. More recently, partly in response to the last financial crisis, Stiglitz [77, 78], Stockhammer [80], and Jayadev [56] among others examine various macro-economic and financial channels for how inequality may contribute to instability and economic and financial crises, with special attention to the case of the US; see Van Treeck [81] for a recent survey.

From the perspective of our theory, a property which distinguishes stratified from egalitarian societies is the degree of fragility versus the robustness of the society. More precisely, our theory suggests a notion of *resilience* that measures the capacity of the society to return quickly to its steady state when subject to any given shock whether demographic, economic, or other.[19] Here *demographic shocks* can include: the spread of an epidemic, a situation which may impose large strains on healthcare facilities; the effect of an earthquake or other natural disaster which involves a large loss of lives and disruption of the economy; military conflicts which inflict a large toll on young individuals in the populations; *economic shocks* can include: a macroeconomic recession or a reform which may lead to a reorganization of the workforce; financial collapse of large banks which can impact a large spectrum of economic projects; a collapse in the price of a commodity, such as oil which is critical for many sectors in the economy and so on.

Casual observation in Europe and other countries across the world reveals a large diversity in the response to shocks. The capacity of the socio-economic communites to remain functional in spite of such shocks seems to be correlated with the degree of economic inequality. To capture this, we first formalize the notion of robustness or resilience and then we state the Perturbation Stability Theorem which implies a negative correlation between the level of economic inequality and the resilience of the society.

Robustness. At a general level, robustness is associated to the invariance of a society's macroscopic variables in the face of endogenous or exogenous shocks or perturbations to the system. The way this is captured by directionality theory is by means of the formalism of large deviation theory (see Demetrius [38] and Demetrius et al. [40]). Formally speaking, our robustness theorem focuses on the macroscopic parameter Φ,[20] and introduces the probability $P_\epsilon(n)$ that sample averages (say $\bar{\Phi}(n)$) of sample trajectories of length n, differ by more than ϵ from ensemble averages over all trajectories, for fixed $\epsilon > 0$. The ergodic theorem states that $P_\epsilon(n)$ converges to zero for large enough sample lengths; moreover, the convergence rate is known to be at least exponentially fast.[21] Hence, as a measure of **robustness** $\mathcal{R}$, we can use the following fluctuation decay rate,

$$\mathcal{R} = \lim_{n\to\infty} \left[-\frac{1}{n} \log P_\epsilon(n) \right]. \tag{18}$$

[19] Aghion, Banerjee, and Piketty [6], Alesina and Perotti [10], and Rodrik [71] have studied models that associate inequality within a society with the political stability of the society. What our model addresses is the speed with which basic economic indicators return to steady state values in the face of any given shock.

[20] As is clear from the Appendix, A.4, this variable is the average of the potential function φ, which is central in determining all other macroscopic parameters of the process $Y(t)$, including the growth rate and the variance. It is for this reason that the robustness measure is based on the probability of sample averages approaching the true value of Φ.

[21] As mentioned in Appendix B.4, it can be shown that there exist constants, $c_0, c_1 > 0$, such that, $P_\epsilon(n) \leq c_0 \exp^{-c_1 n}$.

Large values of $\mathcal{R}$ correspond to fast rates of convergence of the sample averages ($\Phi(n)$) to their steady state values; small values of $\mathcal{R}$ correspond to slow rates of convergence. Thus, $\mathcal{R}$ characterizes the adjustment rate of the (fundamental) macroscopic variable ($\Phi(n)$) in the face of general shocks or perturbations in the underlying system. More generally, because all further macroscopic variables are directly determined through the variable Φ, the measure $\mathcal{R}$ provides a measure of the convergence rate of sample averages of all macroscopic variables to their steady state values. The perturbation stability theorem asserts that changes in robustness are positively correlated with changes in evolutionary entropy, $\Delta H \Delta \mathcal{R} > 0$.

Theorem 4 (Perturbation Stability Theorem)**.** *Societies with higher evolutionary entropy tend to be more resilient to disruptions and perturbations and also tend to return faster to the steady state equilibrium than societies with lower evolutionary entropy.*

As is clear from the proof. It turns out that the entropy measure is a direct measure of the rate of covergence to the steady state. In view of Proposition 1 this can be restated as saying that changes in robustness are negatively correlated with changes in inequality, $\Delta T \Delta \mathcal{R} < 0$. This readily implies the following statement.

Proposition 4 (Robustness and Equality)**.** *Societies with less income inequality tend to be more resilient to disruptions and perturbations and also tend to return faster to the steady state equilibrium than societies with higher income inequality.*

These results point to one significant advantage of more equal societies, namely a typically higher level of resilience to shocks. This has important implications for the fragility or even the expected lifespan or survival rate of societies, Proposition 4 suggests that stratified societies are more likely to be "fragile" in that they are slower in getting back to steady state and they are more likely to be bounded away from their long-run sustainable paths. The opposite holds for egalitarian societies. In the next section we briefly discuss some empirical evidence on this.

7 Empirical Evidence

Assuming certain properties concerning the evolution of the aggregate population and income processes our theory derived several testable implications. While we plan to estimate basic parameters driving the corresponding processes and to test the hypotheses empirically in future work, we here present some evidence for the main results of the paper, taken from the existing literature.

Macroeconomic Volatility and Income Inequality. The central relation derived in the paper relating inequality to the steadiness (and diversity) of the income process (Theorem 2) finds some empirical validation in the literature studying macroeconomic fluctuations and income distribution. Breen and Garcia-Peñalosa [27] study the impact of macroeconomic volatility (measured as the standard deviation of the growth rate of real GDP per capita) on income distribution (measured by the Gini coefficient) for a large set of countries;[22] Laursen and Mahajan [60] study similar effets for different subsets of countries

[22] Our theoretical measure for variability/constancy of the environment is derived from the variable Φ, which measures the difference between the steady state growth rate and the rate at which the macroscopic variables return to steady state values; the larger Φ the more erratic the process and the larger its volatility measured in terms of the standard deviation of the year to year growth rate. An economy with a very low (negative) Φ will exhibit a year to year growth rate typically very close to its steady state value; while one with a large (positive) Φ will exhibit a less stable year to year growth rate. In the absence of an empirical measure for Φ we think the volatility measure used in this and other papers are reasonable proxies. In Appendix A.4 we also formally show a positive correlation of Φ with the measure of volatility σ^2.

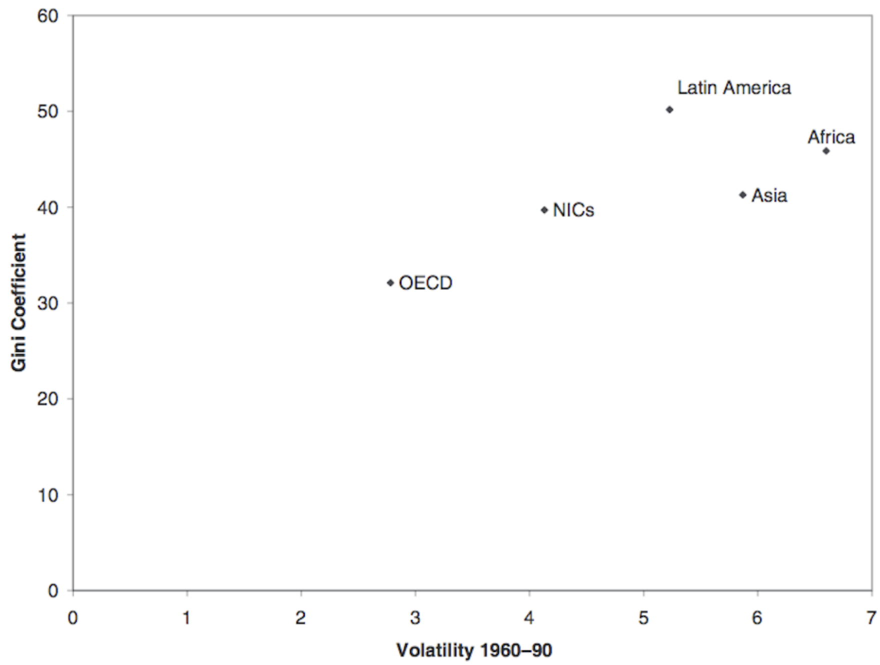

Figure 3: Income inequality and macroeconomic volatility, averaged by region, [27].

using the income share of the bottom quintile of the population as inequality measure. More recently, Huang et al. [55] study macroeconomic fluctuations and inequality across the US. All these papers find a significant and positive correlation between macroeconomic volatility and income inequality, which is consistent with our theory, linking, as a first-order effect, fluctuations of the environment to fluctuations of aggregate production, through the evolutionary entropy, and this in turn to income inequality. Neither paper looks at the role of the diversity or singularity of the income process; and neither paper estimates the interaction matrix or the parameters Φ and γ. Such a systematic study of these relationships is deferred to future work. Figure 3, taken from Breen and Garcia-Peñalosa [27], plots income inequality on volatility.

In the paper, we have argued that our notion of evolutionary entropy which is at the center of our formal derivations is also a measure of the level of cooperation of society. Thus we expect societies with high volatility to exhibit low levels of cooperation, high levels of selfishness, and societies with low volatilies to exhibit high levels of cooperation, high levels of altruism or mutualism. General measures of trust have been found to be closely connected to measures of cooperation (see e.g., La Porta et al. [59]). Consistent with our approach (in particular with the Entropic Selection Theorem linking evolutionary entropy to the volatility of the environment) is the finding of a significant and negative relationship between trust and macroeconomic volatility in Sangnier [73]. Figure 4 taken from Sangnier [73] plots macroeconomic volatility on trust, which we take as a proxy for the level of cooperation in the society.[23] It is also worth pointing out that, again consistent with our approach, trust and indices of social capital have been found to be negatively correlated with income inequality (e.g., Putnam [69] and Wilkinson and Pickett [83]).

[23] The measure of trust used here is given by the share of people who answered *"most people can be trusted"* to the following question of the World Values Survey between 1981 and 2008: *"Generally speaking, would you say that most people can be trusted or that you need to be very careful in dealing with people?"*, Sangnier [73], p. 653.

Figure 4: Macroeconomic volatility and trust, by country, [73].

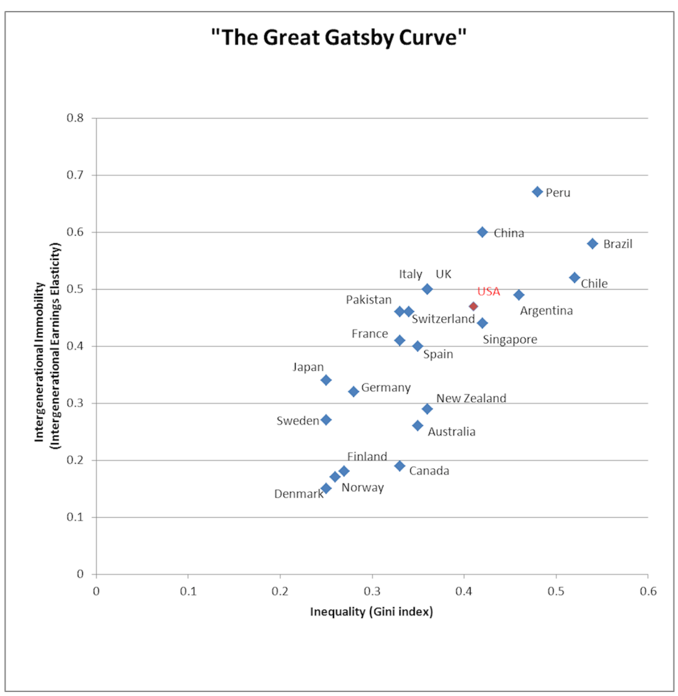

Figure 5: Income immobility and income inequality, by country, [32].

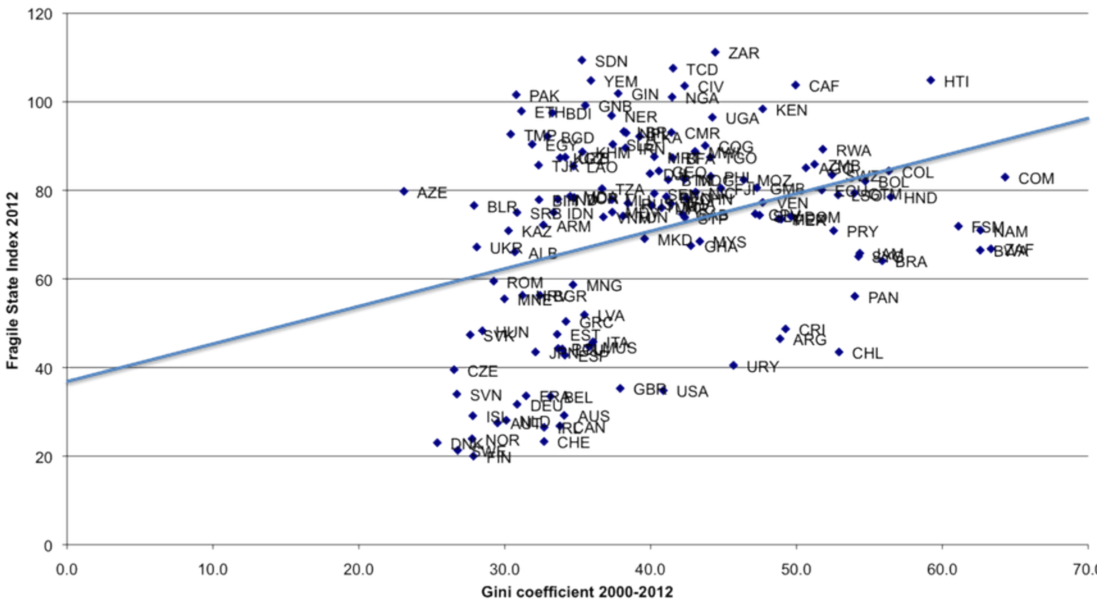

Figure 6: State fragility and income inequality, by country, [FFP and World Bank].

Social Mobility and the Great Gatsby Curve. The *Great Gatsby Curve*, plotting intergenerational income mobility on income inequality, was highlighted recently in a speech delivered by the chairman of the US Council of Economic Advisors, Alan Krueger. He pointed to a positive relation between income inequality (measured by the Gini coefficient) and intergenerational immobility (measured by the intergenerational income elasticity) in the United States and in twelve developed economies; see the 2012 US Economic Report of the President [32], and also Andrews and Leigh [13], Blanden [22], and Corak [31]; Chetty et al. [30] find a negative correlation between inequality and upward mobility in the US. This is very much consistent with our theory that associates steady and diverse environments to both higher income equality and higher income and social mobility; and associates fluctuating and singular environments to higher inequality and lower income and social mobility (Theorems 2 and 3). In particular, income inequality will be negatively correlated with measures of income mobility; Figure 5, taken from 2012 US Economic Report of the President [32] plots intergenerational income elasticity (a measure of income immobility) on income inequality and finds a positive correlation. The relationship has also been obtained in the labor economics literature with the earnings returns to human capital and the "progressivity" of public investment in human capital as key links; see Solon [74].

Inequality and Resilience to Shocks. The role of inequality in affecting the resilience of a country to respond to shocks was an important topic of a 2011 report of the United Nations Development Programme. In the report the authors point out that indeed less equal societies are slower in adapting to shocks; they further show that these are also the countries with less stable environments. Anbarci et al. [12] and Kahn [57] study respectively earthquakes and natural disasters and show that besides national income, the level of income inequality has an important effect on the death toll and fatalities; Alesina and Perotti [10] show that income inequality fuels discontent and is empirically positively correlated with political instability;

see also Rodrik [71].

One can view the "fragility" of a country as the inverse of our notion of robustness. The Fund for Peace (FFP) computes an index they call the *Fragile State Index* which is an average of a dozen indicators measuring a variety of economic, social, political, and military aspects of a country that are meant to capture its overall level of "fragility". Based on their index, they categorize countries from being sustainable (e.g., Scandinavian countries typically rank at the top) to being in a state of high alert (many countries in Africa, Asia, Latin America and the Middle East are in this category). In Figure 6 we plot the *Fragile State Index* for 2012 on the corresponding Gini coefficient for over 130 countries (averaged over the years 2000-2012; taken from World Bank data); we have $R^2 = 0.104$. The observed significant and positive correlation suggests that indeed more unequal countries also tend to be more fragile. This is roughly consistent with our perturbation stability result (Proposition 4), whereby more equal societies are faster in adjusting to shocks and therefore also more likely to be in a sustainable state and less likely to be in or close to a state of alert.

8 Conclusion

Discussions regarding the phenomenon of economic inequality, its origin and spread, have moved recently from the confines of specialized academic departments to being among the most debated topics of the general public. At least three issues appear to fuel the debate, namely, the empirical reality that the gap between rich and poor has shown a remarkable increase in a number of countries in the last 30 years (Piketty [68]); the empirical observation that countries with large economic inequality have a high degree of dysfunction in a number of canonical indices, including health, corruption, and crime as well as economic and political stability (Acemoglu et al. [3], Acemoglu and Robinson [5], Stiglitz [78], Wilkinson and Pickett [83]); the fragility and instability of highly stratified societies often generates flows of immigrants that in turn creates further problems for hosting countries.

The theory developed in this paper contributes to the understanding of the origin and spread of inequality by recognizing the importance of the macroeconomic environment in driving income distribution. The intermediary element connecting the macroeconomic environment and the income distribution are the social preferences or cooperative dispositions of the individuals in the society. The theory shows that steady and diverse environments induce more equal redistribution, mediated through a high degree of cooperativeness (high evolutionary entropy); while fluctuating and singular environments favor less equal redistribution, mediated through a high degree of selfishness (low evolutionary entropy).[24] Furthermore, the environment, through its effect on cooperativeness and the evolutionary entropy, will also impact both intergenerational social mobility and the resilience of the economies in such a way that reinforces and amplifies the original effect, thus explaining the emergence and persistence of egalitarian societies on one side and highly stratified ones on the other.

[24] Some have raised the question of how our model can explain the drastic increase in inequality in the US or the UK in recent decades. To answer this question, one would need to estimate the corresponding interaction matrices and derive the empirical parameters Φ and γ. It would be interesting to check whether one can observe a structural break in the parameters around roughly 1980 with a markedly increased value for Φ. To the extent that for important segments of the population, levels of uncertainty have increased (see Bourguignon [24], and see Stiglitz [78] for the US); and to the extent that deregulation and liberalization (and also globalization) have lead to increases in volatility for key sectors of the economy, our model might indeed predict an increase in inequality and a decrease in upward mobility. This is not inconsistent with the fact that the increase in inequality seems to be driven to a significant extent from top incomes (Bourguignon [24], Piketty [68], Stiglitz [78]); especially if these incomes are derived from sectors, such as the financial sector, that have been subject to important deregulation and volatility. Clearly, more work is required to test empirically what our theory can contribute to this debate; this may also shed light on the stark decrease in "social capital" documented for the US by Putnam [69].

Regulation, fiscal, and monetary policy may be designed to take this into account. Indeed the theory provides a new rationale for macroeconomic stabilization policies. The case for such policies has been rather discredited by new classical models of macroeconomics, with Lucas famously putting the cost of fluctuations at "something less than one tenth of a percentage point" of average consumption (Lucas [62], p. 27; also see Krusell et al. [58] for a more recent revised estimate). On the other hand, consistent with our viewpoint, the German law of 1967, "*Gesetz zur Förderung der Stabilität und des Wachstums der Wirtschaft*," *(StabG)*, explicitly identifies the macro-economic stability as a prime objective of economic policy and public finance;[25] in a similar sense the United States "*Full Employment and Balanced Growth Act of 1978*" strengthens the preceding "*Full Employment Act of 1946.*" Our theory shows that policies that can ensure a steady and diverse economy can contribute to raising the level of cooperation, trust, equality, and mobility in the society, which in turn will contribute to improving its resilience and robustness. At the same time, policies that attempt to address issues of, say equality and mobility, without taking into account the steadiness and diversity of the environment may fail to achieve their intended objectives.

In his recent book, Piketty [68] documents high and increasing levels of concentration of income and wealth in several industrialized countries and calls for strong taxation, including on wealth, to reverse the trend. From the viewpoint of this paper, such strong taxation may not be effective if the macroeconomic environment or the underlying income generating process does not exhibit sufficient steadiness and diversity to be compatible with the necessary cultural background that would ensure compliance to strong taxation on the part of society.[26] An unsteady and non-diverse macroeconomic environment, by confering a selective advantage to self-serving behavior, will act as a force going against the objective of redistribution. Thus when designing policies with the objective to redistribute resources, policy makers should monitor the steadiness and diversity of the underlying income-generating process. Clearly, more empirical work is needed to test the different mechanisms and correlations derived in this paper. Hopefully this will provide a fuller understanding of the role of macroeconomic volatility and diversity in long-term macro-processes that will allow to better design and evaluate economic policies.

[25] Consistent with this, former Chancellor Helmut Schmidt elevated the objective of a stable economy to a cornerstone of West Germany's national and international strategic policy, with the further objective of exporting such a policy to neighboring European countries (see, e.g., his speech of November 30, 1978, to the Bundesbank's board of directors). With a Gini coefficient of 0.25, West Germany exhibited relatively low inequality throughout the 1980's before unification in 1990.

[26] The proliferation of a shadow economy or of fiscal paradises could perhaps be seen as a consequence of volatile environments combined with levels of taxation that are excessive relative to the country's cultural background. Clearly, this calls for further empirical research.

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APPENDIX

A Background Analysis

We here build on Arnold et al. [14] and Demetrius and Gundlach [39] to sketch some basic properties of the dynamic system and of its macroscopic variables at steady state. This allows us to better understand the terminology and connections between the different variables in the model. These facts are derived in detail in the cited articles using the formalism of random dynamical systems and statistical mechanics. While we can only limit ourselves to sketching the main steps, we refer to those article for a complete discussion; see also Demetrius [38] and Demetrius et al. [41] for further discussion. We start with a short example of a demographic model from population biology that best illustrates the source of the stochasticity of our underlying model.

A.1 An Illustrative Example: The Leslie Matrix

Consider the evolution of a population divided into d age classes. Let $N(t) = \sum_{i=1}^{d} n_i(t)$ denote the total (finite) population size, where $n_i(t)$ is the number of individuals in age class i in period t. A natural way of representing the evolution of the population is by means of the equation,

$$n(t+1) = Bn(t),$$

where $n(t) = (n_1(t), \ldots, n_d(t))$ is a d-vector and B is the $d \times d$ *Leslie matrix* defined by,

$$B = \begin{pmatrix} m_1 & m_2 & \cdots & \cdots & m_d \\ b_1 & 0 & \cdots & \cdots & 0 \\ 0 & b_2 & \ddots & \cdot & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & b_{d-1} & 0 \end{pmatrix},$$

where m_i is the *reproduction rate* of an individual in age-class i (this is the mean number of offspring produced by an individual in age class i), and b_i is the *survival rate* of individual in age class i (this is the proportion of individuals that survive from age class i to age-class $i+1$). The above law of motion describes the steady state of the system. However, there is a natural stochasticity underlying this system which derives from the finiteness number of individuals and the fact a single individual cannot have, say, 1.3 offspring, but rather, either 0, 1, 2, or more. The exact number of offspring of an individual or whether or not he/she survives to the next age class is modeled as a random integer-valued event that gives rise to a branching process that under regularity conditions can be shown to lead to a CIR-type process with the parameters r and σ^2 as mentioned in Section 2.

A.2 Random Dynamical Systems

We assume that the (possibly nonlinear) dynamic system,

$$v(t+1) = A(t)v(t)$$

evolves to a steady state. At steady state we assume the process is represented living a constant $d \times d$ matrix $A = (a_{ij})$ with $a_{ij} > 0$. As explained above, the entries a_{ij} are rates that represent averages. In fact, transactions between one class and another are assumed to occur in discrete units and with fixed probabilities. Let $D = \{1, 2, \ldots, d\}$ and define the set of all possible infinite backward sequences

$$X = \prod_{\nu=-\infty}^{\infty} D_\nu, \qquad \text{where } D_\nu = D.$$

We can then define the space of all possible infinite backward sequences

$$\Omega = \{x \in X : a_{x_{\nu+1}x_\nu} > 0\},$$

which we also refer to as the interaction space. It represents the space of all infinite backward paths of the graph associated with the matrix A. These can be thought of following through the past transitions of a given "representative" unit of the numeraire commodity between different individuals of different classes.

Let $\tau : \Omega \to \Omega, (x_k) \mapsto (\tilde{x}_k)$, where $\tilde{x}_k = x_{k+1}$ be the shift map, and let $\mathcal{M}$ denote the set of probability measures that are invariant under the shift map τ. Defining μ as the natural Markov measure on Ω at the steady state, one can show that (see [41], Theorem 4.2) this is the unique probability measure that maximizes $H_\mu(\tau) + \int \varphi d\mu$ such that,

$$\log \lambda = \sup_{\mu \in \mathcal{M}} \left\{ H_\mu(\tau) + \int \varphi d\mu \right\}, \tag{19}$$

where $H_\mu(\tau)$ is the Kolmogorov-Sinai entropy for the system (Ω, μ, φ), and the function $\varphi : \Omega \to \mathbb{R}$ is given by

$$\varphi(x) = \log a_{x_1 x_0}.$$

Analytically, it can be described explicitly by means of the Markov matrix $P = (p_{ij})$, where $p_{ij} = \frac{a_{ij}v_j}{\lambda v_i}$, and $v = (v_i)$ is the right eigenvector corresponding to the largest eigenvalue λ of the matrix A; (see Arnold et al. [14] and also Demetrius et al. [41], Theorem 4.2).

Furthermore, one can show using a generalization of the Central Limit Theorem (see Demetrius et al. [41], Theorem 7.1) that, under the Markov measure μ, for the dynamical system defined by Eq. (4), asymptotically, the deviations from the mean of appropriately constructed sample paths, as $n \to \infty$, can be approximated by a Brownian motion with variance $\sigma^2 t$. Taking time limits, this yields a continuous time process with growth rate r and variance $\sigma^2 Y(t)$ that can be represented as the diffusion given by Eq. (7); (see Demetrius et al. [41], Section 7); the process belongs to the class of so-called Cox-Ingersoll-Ross processes studied in mathematical finance; see Cox et al [34].

A.3 Perturbations

Throughout the paper we make use of perturbations of the interaction matrix of the form $A(\delta) = A^{1+\delta}$ that capture the variations or deviations from the steady state of the intertemporal production process (4). We here sketch a motivation for the specific one-parameter form we adopt throughout the text.

Consider two dynamic systems at steady state given respectively by (Ω, μ, φ) and $(\Omega, \mu^*, \varphi^*)$, where to capture the fact that say the latter is a variation (mutation) of the former, we assume that

$$\varphi^* = \varphi(\delta) = \varphi + \delta\psi,$$

satisfying the conditions

$$\int \varphi d\mu = \int \psi d\mu \quad \text{and} \quad \frac{d}{d\delta}\int \varphi d\mu \,|_{\delta=0} \;=\frac{d}{d\delta}\int \psi d\mu \,|_{\delta=0}\;.$$

The first condition says that the deviation ψ of the variant (mutant) population has the same reproductive potential as the incumbent population; the second condition says that it changes in the same direction. This is sufficient for our results. However, for ease of representation and to simplify the analysis we consider the special case $\psi = \varphi$, which, if we assume $\varphi = \log a_{ij}$ (for corresponding realization $a_{x_1x_0} = a_{ij}$), implies

$$\varphi(\delta) = \varphi + \delta\varphi = (1+\delta)\log a_{ij} = \log a_{ij}^{1+\delta},$$

which corresponds to perturbations of the interaction matrix of form

$$A(\delta) = (a_{ij}^{\delta}) \equiv A^{1+\delta},$$

considered throughout the paper. A perturbation where $\delta > 0$ ($\delta < 0$) corresponds to a variant population with lower (higher) entropy H_{μ^*} as compared to the entropy of the resident population H_μ, where $\delta = 0$. See Demetrius et al. [41] for further discussion.

A.4 Macroscopic Variables

Applying statistical mechanics tools to the dynamical system described in the previous section shows how to derive macroscopic variables from the function φ and the measure μ. These allow a more concise description of the dynamical system involved.

Growth Rate. Consider the function

$$S_n\varphi(x) = \sum_{k=0}^{n-1}\varphi(\tau^k x) = \sum_{k=0}^{n-1}\log a_{x_{k+1}x_k},$$

so that denoting for given $(x_0, x_1, \ldots, x_n)$ by x^* any point in Ω with $x_i^* = x_i$, for $i = 0, 1, \ldots, n$, then we have

$$Z_n(\varphi) = \sum_{(x_0,x_1,\ldots,x_n)} \exp S_n\varphi(x^*) = \sum_{(x_0,x_1,\ldots,x_n)} a_{x_1x_0}a_{x_2x_1}\cdots a_{x_nx_{n-1}},$$

so that, using the Perron-Frobenius Theorem, in the limit we have

$$\lim_{n\to\infty}\frac{1}{n}\log Z_n(\varphi) = \log\lambda = r,$$

which exists under general conditions on φ. From the variational principle (19) mentioned above, we have

$$r = \Phi + H,$$

where $H = H_\mu(\tau)$ denotes the Kolmogorov-Sinai entropy of (Ω, μ, φ) and $\Phi = \int \varphi d\mu$ denotes the reproductive potential or the mean energy associated with the (potential) function φ; see Arnold et al. [14], Demetrius [38].

Reproductive Potential and other Moments. From the growth rate we can generate further macroscopic variables. Let $\lambda(\delta)$ denote the dominant eigenvalue of the perturbed matrix $A(\delta) = A^{1+\delta} = (a_{ij}^{\delta})$, and $r(\delta) = \log \lambda(\delta)$, for $\delta \in \mathbb{R}$. Then as shown in Demetrius et al. [41], we have,

$$r(\delta) = r(0) + \delta r'(0) + \frac{\delta^2}{2!} r''(0) + \frac{\delta^3}{3!} r'''(0) + \dots,$$

where

$$\begin{aligned} r'(0) &= \lim_{n\to\infty} \frac{1}{n} \mathbb{E}_n \left[S_n \varphi \right] = \int \varphi d_\mu = \Phi \\ r''(0) &= \lim_{n\to\infty} \frac{1}{n} \mathbb{V}_n \left[S_n \varphi \right] = \sigma^2 \\ r'''(0) &= \lim_{n\to\infty} \frac{1}{n} \mathbb{E}_n \left[S_n \varphi - \mathbb{E}_n S_n \varphi \right]^3 = \kappa. \end{aligned}$$

Here $\mathbb{E}_n$ and $\mathbb{V}_n$ denote the expectation and variance with respect to the measure μ_n on finite sequences of length n of the form $(x_0, x_1, \dots, x_n)$, which is defined by

$$\mu_n = \frac{S_n \varphi(x)}{\sum_{(x_0, x_1, \dots, x_n)} S_n \varphi(x)}.$$

Besides giving the reproductive potential Φ, this further gives the demographic variance σ^2, and the correlation index κ; clearly we can also write $\Phi = \frac{dr(\delta)}{d\delta} \left.\right|_{\delta=0}$.

Consider now the perturbed variance $\sigma^2(\delta)$ obtained by further perturbing $A(\delta)$ again. Then we can define

$$\gamma = \frac{d\sigma^2(\delta)}{d\delta} \left.\right|_{\delta=0},$$

which is referred to as the demographic index; (as it can be used to approximate the demographic variance).

Relations of the Macroscopic Variables. We here summarize some of the relationships that hold between the macroscopic variables defined so far; (see Demetrius et al. [41] for derivations):

(a) $\Phi = r - H$

(b) $\gamma = 2\sigma^2 + \kappa$.

In our evolutionary analysis, an incumbent population is in competition with a variant (or invader) population, which we capture in terms of a dynamic interaction between the two populations. The incumbent and the variant (invader) population steady state dynamics are given respectively by (Ω, μ, φ) and $(\Omega, \mu^*, \varphi^*)$, where to capture the fact that the latter is a mutation of the former, as discussed in A.3, we assume that

$$\varphi^* = \varphi(\delta) = \varphi + \delta\varphi,$$

which corresponds to an interaction matrix of the invader population of the form $A(\delta) = A^{1+\delta}$.

We can then determine the macroscopic variables for the variant population as with the incumbent population, so that setting $r^* = r(\delta)$, ${\sigma^*}^2 = \sigma^2(\delta)$, and $H^* = H(\delta)$, we get:

(c) $\Delta r = r(\delta) - r(0) \approx \Phi\delta$

(d) $\Delta\sigma^2 = \sigma^2(\delta) - \sigma^2(0) \approx \gamma\delta$

(e) $\Delta H = H(\delta) - H(0) \approx -\sigma^2\delta$.

For small $\delta \in \mathbb{R}$, this readily gives the following relations:

(f) $\Phi < 0 \iff \Delta r \Delta H > 0$

(g) $\gamma > 0 \iff \Delta\sigma^2 \Delta H < 0$.

These will play an important role in the derivation of the main results.

Interaction between Environment, Population, and Production. Throughout the paper we refer to the variable Φ as representing the constancy of the environment or of the income process. This was motivated in Section 2 by referring to the relationship $\Phi = r - H$. In Section 7, discussing the empirical evidence, we also associated Φ loosely to what is called macroeconomic volatility. We here make the connection more formal by showing that it is positively correlated to σ^2,

(h) $\Delta\sigma^2 \Delta\Phi > 0$,

for $\delta \in \mathbb{R}$ small and for $\Phi > -\sigma^2$ and $\gamma > 0$. To see this, notice that $\Delta\sigma^2\Delta\Phi \approx \gamma(\Phi + \sigma^2)\delta^2$. While this does not cover all parameter values, we believe it covers most cases of empirical relevance; especially the case of $\gamma < 0$ seems empirically-speaking quite rare.

Environmental variables are in correspondence with both population and production variables, so that changes in first-order and second-order moments are strongly positively correlated at the steady state equilibrium. This plays a central role in linking our resource process to both the population and the production process. We capture this by assuming,

(i) $\Delta r \Delta\widetilde{r} > 0$,

(j) $\Delta\sigma^2 \Delta\widetilde{\sigma}^2 > 0$,

which in particular implies,

(k) $\Delta H \Delta\widetilde{H} > 0$.

These relations play a role in establishing the link between social mobility and income inequality in this framework.

A.5 Evolution of Cooperation and Evolutionary Entropy

In the biology literature, the analytic study of the evolution of cooperation goes back to Hamilton's model [54], called the theory of inclusive fitness (or kin selection); see also Davies et al. [35] and Bourke [25] for textbook treatments. This model considers the gene as the unit of selection. The evolution of social behavior is analyzed in terms of the genes for behavioral traits such as altruism and selfishness. The fundamental parameter is the coefficient of genetic relatedness between the actor and the recipient. The central result is *Hamilton's rule*, which asserts that the evolution of altruism and selfishness can be qualitatively described in terms of constraints in the parameter of genetic relatedness, such that, when genetic relatedness is *high*, altruism will have a selective advantage and will increase in frequency, and, when genetic relatedness is *low*, selfishness will have a selective advantage and will increase in frequency.

Inclusive fitness theory has had a significant impact on the study of sociality and cooperation in social insects and has achieved a canonical status in studies of organisms where the effect of genetic relatedness can be readily observed. The theory has been extended beyond the narrow model of kin selection to include wider forms of cooperation. Lehman and Keller [61] review the literature on the evolution of cooperation providing a classification of necessary conditions that may induce cooperative or altruistic behavior; which include, kin selection, direct benefits from cooperation, and reciprocation and enforcement through repeated interaction. Some of these mechanisms have also been studied in the economics literature, as discussed in Bowles and Gintis [26]. However, critics of the theory, for example, Bowles and Gintis [26] and Wilson [84], have emphasized that there are observed forms of cooperation and altruism in humans that extend beyond close genealogical kin or reciprocity in repeated interactions to include cooperation with even total strangers.

The entropic theory of sociality described in Demetrius and Gundlach [39] is a general theory of cooperation which accommodates the evolution of many kinds of social behavior. It revolves around a relation between evolutionary entropy and cooperation which is embodied in the following *entropy-sociality rule*: Evolutionary entropy parameterizes the level of cooperation of a population along an axis such that *high* entropy corresponds to altruism, mutualism or a high degree of cooperation and *low* entropy corresponds to selfishness or a low degree of cooperation.

The nexus of the theory is the incorporation of environmental or resource constraints and demographic constraints in the model. The model thus distinguishes two classes of resource-demographic constraints: (i) Limited resources and finite population size; (ii) Unlimited resources and large, effectively infinite, population size. When (i) holds, the theory predicts that the evolution of cooperation will be contingent on resource and demographic constraints; this is where the variability and diversity of the resources enter into the theory. When (ii) holds, the theory predicts that the evolution of cooperation will be contingent on the degree of relatedness between the individuals or the groups. In this case, the evolutionary rules admit the following qualitative cases: ($ii.1$) When relatedness is *high*, mutualism will have a selective advantage and increase in frequency; ($ii.2$) When relatedness is *low*, selfishness will have a selective advantage and increase in frequency.

These results show that the entropic theory of cooperation is related to the inclusive fitness theory and, in particular, Hamilton's rule, mentioned above, as follows: Hamilton's rule is obtained as the limit of resource abundance and population approaching infinity, $X \to \infty$ and $N \to \infty$, of the entropy-sociality rule. Thus, Hamilton's rule, which ignores the effects of resource and demographic constraints, can be seen as an approximation to the entropic theory of cooperation, which improves in accuracy as resources and population sizes increase. See Demetrius and Gundlach [39], Section 8.

Evolutionary Entropy and Cooperation. For the sake of concreteness, we briefly sketch how the general notion of evolutionary entropy H used throughout the paper can be related to a simple notion of cooperation. We refer to Demetrius and Gundlach [39], Section 7, for a fuller treatment. Consider the interaction between two individuals, $i = 1, 2$. To make things particularly simple, we assume the interactions are symmetric, meaning that we can write the interaction matrix (representing the interaction between these two individuals) in the form:

$$A = \begin{pmatrix} a & b \\ b & a \end{pmatrix},$$

where $a > 0$ measures resource allocation to self, and $b > 0$ measures resource allocation to non-self. Suppose we write $b = \rho a$, where $\rho \in (0, 1)$ is a parameter measuring the degree of *cooperativeness* or reciprocity in the interaction. Thus, if $\rho = 0$ there is low or no cooperation, and if $\rho = 1$ there is high cooperation. Then, using our formula for the evolutionary entropy, we can compute,

$$H = -\frac{2}{1+\rho} \log \frac{1}{1+\rho} - \frac{2\rho}{1+\rho} \log \frac{\rho}{1+\rho}.$$

This expression is maximized at $\rho = 1$, that is, at high cooperation, where $b = a$ (fully mutuakistic behavior), and it is minimized at $\rho = 0$, that is, at low cooperation, where $b = 0$ (fully selfish behavior). This can be extended to the case of a larger population of individuals divided in $d \geq 2$ classes.

B Proofs

B.1 Proof of Theorem 1

We here povide a sketch of the main steps. For a more detailed proof, we refer the reader to Demetrius et al. [41].

Let $Z(t) = Y(t) + Y^*(t)$ denote total aggregate production. The share of aggregate production of the invader population can be written as, $p(t) = \frac{Y^*(t)}{Z(t)}$. We are concerned with the evolution of this ratio.[27] The densities of $f(Y, t)$ and $f^*(Y^*, t)$ of the aggregate production levels $Y(t)$, and $Y^*(t)$ are characterized respectively by the Fokker-Planck equations

$$\frac{\partial f}{\partial t} = -r\frac{\partial(fY)}{\partial Y} + \frac{\sigma^2}{2}\frac{\partial^2(fY)}{\partial Y^2} \quad \text{and} \quad \frac{\partial f^*}{\partial t} = -r^*\frac{\partial(f^*Y^*)}{\partial Y^*} + \frac{{\sigma^*}^2}{2}\frac{\partial^2(f^*Y^*)}{\partial {Y^*}^2}.$$

Equivalently, we can characterize $Y(t)$ and $Y^*(t)$ respectively as the solutions to the stochastic differential equations

$$dY(t) = rY(t)dt + \sigma\sqrt{Y(t)}dW(t), \tag{20}$$

and

$$dY^*(t) = r^*Y^*(t)dt + \sigma^*\sqrt{Y^*(t)}dW^*(t), \tag{21}$$

where the processes $Y(t)$ and $Y^*(t)$ evolve simultaneously and stochastically independently, so that $W(t)$ and $W^*(t)$ are independent Brownian motions.

Let $A^* = A(\delta) = A^{1+\delta}$ represent the interaction matrix of the variant population with coresponding macroscopic variables $r^* = r(\delta), {\sigma^*}^2 = \sigma^2(\delta), H^* = H(\delta), \Phi^* = \Phi(\delta), \gamma^* = \gamma(\delta), \kappa^* = \kappa(\delta)$.

It can be shown (see Demetrius et al. [41], Theorem 7.2) that equations (20) and (21) are equivalent

[27] Initially, the share $p(t)$ is small and the two populations evolve independently of each other. The invader population can be seen as drawing from resources not used or available to the incumbent. Then, as the invader population grows, the two populations compete for resources. We assume the two populations are in steady state assuming indirectly that the convergence to steady state is much faster than the selection process. This also justifies focusing on the case where the overall production is fixed ($Z(t) = Y$); see also Demetrius et al. [41], Section 2.

to the system of stochastic differential equations,

$$dZ(t) = (r + p(t)\Delta r)\, Z(t)dt + \sigma\sqrt{(1 - p(t))Z(t)}dW(t) + \sigma^* \sqrt{p(t)Z(t)}dW^*(t), \tag{22}$$

and

$$dp(t) = p(t)(1 - p(t)) \left(\Delta r - \frac{\Delta\sigma^2}{Z(t)}\right) dt - \sigma p(t)\sqrt{\frac{(1 - p(t))}{Z(t)}}dW(t) + \sigma^*(1 - p(t))\sqrt{\frac{p(t)}{Z(t)}}dW^*(t). \tag{23}$$

We need to solve this for the process $p(t)$. Assuming total aggregate production is constant, $Z(t) = Y$,[28] then the process $p(t)$ can be shown to be a diffusion process with drift

$$\alpha(p(t)) = p(t)\,(1 - p(t)) \left(\Delta r - \frac{\Delta\sigma^2}{Y}\right)$$

and variance

$$\beta(p(t)) = \frac{p(t)(1 - p(t))}{Y} \left(\sigma^2 p(t) + {\sigma^*}^2(1 - p(t))\right);$$

and that the process $p(t)$ has density ψ solving the Fokker-Planck equation (see Demetrius et al. [41], Theorem 7.3),

$$\frac{\partial\psi}{\partial t} = -\frac{\partial[\alpha(p)\psi]}{\partial p} + \frac{1}{2}\frac{\partial^2[\beta(p)\psi]}{\partial p^2},$$

with natural boundary conditions, $\psi(0, t) = 0, \psi(1, t) = 1$, that correspond to the cases $p = 0$ (when the variant population becomes extinct) and $p = 1$ (when the incumbent population becomes extinct). Notice that we set $\alpha(p) \equiv \alpha(p, Y)$ and $\beta(p) \equiv \beta(p, Y)$, so that $\alpha(0) = \alpha(1) = 0$ and $\beta(0) = \beta(1) = 0$. This implies a unique solution for any initial value $\psi(p, 0)$.

Letting $p_0 = p(0)$ denote the initial frequency of the mutant and letting $\rho(p_0)$ denote the probability that the diffusion process leads to an absorption in the state $p = 1$ (extinction of the incumbent population), appealing to the backward Kolmogorow equation,

$$\frac{\partial\psi}{\partial t} = \alpha(p)\frac{\partial\psi}{\partial p} + \frac{1}{2}\beta(p)\frac{\partial^2\psi}{\partial p^2}$$

and integrating, one shows that the invasion probability $\rho(p_0)$ can be written as,

$$\rho(p_0) = \frac{1 - \left(1 - \frac{\Delta\sigma^2}{{\sigma^*}^2}p_0\right)^{\frac{2YS}{\Delta\sigma^2}+1}}{1 - \left(1 - \frac{\Delta\sigma^2}{{\sigma^*}^2}\right)^{\frac{2YS}{\Delta\sigma^2}+1}},$$

where $S = \Delta r - \frac{\Delta\sigma^2}{Y}$ (again, see Demetrius et al. [41], Section 7). The sign of the expression S thus becomes crucial in determining whether a variant is successful in invading or not. Except for the degenerate case of $\frac{2YS}{\Delta\sigma^2} + 1 = 0$, we have $\rho'(\cdot) \neq 0$, and it is easy to show that convexity or concavity of $\rho(\cdot)$ is determined by S alone, namely,

$$S > 0 \Rightarrow \rho(\cdot) \text{ is convex}, \qquad S < 0 \Rightarrow \rho(\cdot) \text{ is concave}.$$

[28] Strictly speaking, we need only to assume that this holds for $t > t_0$ for some t_0 that represents the instant where the exploitation competitive interaction between incumbent and invader population begins; this is consistent with the case we consider, where resources are finite and limited. See Demetrius et al. [41], Section 2, for further discussion on this point.

The exact curvature of $\rho(\cdot)$ then depends on the magnitude of S and hence on the values of $\Delta r, \Delta\sigma^2$, and Y. The exact relations between these variables in determining the sign of S and their effect on the invasion probability provides the conditions under which an invader's level of entropy should be higher or lower than H in order to be successful.

Now, consider initial values p_0 close to zero, then the solution $p(t)$ is absorbed in state $p = 0$ (extinction of the invader population) for any small perturbation, if

$$\Delta r < 0, \Delta\sigma^2 \geq 0 \qquad \text{or} \qquad \Delta r \leq 0, \Delta\sigma^2 > 0. \tag{24}$$

Under these conditions, one of the following two cases occurs,

(I) $\Phi < 0$, $\gamma \geq 0$, or $\Phi \leq 0$, $\gamma > 0$;

(II) $\Phi > 0$, $\gamma \leq 0$, or $\Phi \geq 0$, $\gamma < 0$.

In case *(I)*, condition (24) for all perturbations is equivalent to $\Delta H < 0$ (the variant population has lower entropy; and the incumbent population with higher entropy takes over; recall that, $\Phi < 0 \Rightarrow \Delta r \Delta H > 0$ and $\gamma > 0 \Rightarrow \Delta\sigma^2\Delta H < 0$); in case *(II)*, it is equivalent to $\Delta H > 0$ (the variant population has higher entropy; and the incumbent population with lower entropy takes over; recall that, $\Phi > 0 \Rightarrow \Delta r \Delta H < 0$ and $\gamma < 0 \Rightarrow \Delta\sigma^2\Delta H > 0$); (see [41], Theorem 7.4). This yields the more general formula for the selective advantage

$$S = -\left(\Phi - \frac{\gamma}{Y}\right)\Delta H,$$

where $\Delta H = H^* - H$.

In the limit, as $Y \to \infty$, the diffusion equation for p degenerates to a linear differential equation, and the convexity criterion in terms of S reduces to the growth rate differential Δr. In this case, we have, $\Phi < 0 \iff \Delta H < 0$ and $\Phi > 0 \iff \Delta H > 0$, and the reproductive potential alone determines the selective advantage of the level of entropy (again, see Demetrius et al. [41], Section 7).

B.2 Proof of Proposition 1

Consider the following further measure of entropy,

$$H_Y = -\sum_{i=1}^{d} s_i \log s_i, \qquad 0 \leq H_Y \leq \log d,$$

where $s_i = v_i / \sum_{j=1}^{d} v_j$ is the steady state share of income of class i. Then it is easy to see that H_Y and $T(v)$ are related as follows,

$$T(v) = \log d - H_Y,$$

so that $\Delta H_Y \Delta T < 0$.

We need to show that $\Delta H_Y \Delta H > 0$. To see this, consider the following matrix

$$A_Y = \begin{pmatrix} v_1 & \cdots & v_1 \\ \vdots & & \vdots \\ v_d & \cdots & v_d \end{pmatrix}$$

where v is the right eigenvector of A. Then it is easy to see that it is also a right eigenvector of A_Y,

$$A_Y v = \lambda_Y v,$$

where $\lambda_Y = \sum_{i=1}^d v_i$. Moreover, the corresponding Markov matrix takes the form

$$P_Y = \begin{pmatrix} \frac{v_1}{\lambda_Y} & \cdots & \frac{v_d}{\lambda_Y} \\ \vdots & & \vdots \\ \frac{v_1}{\lambda_Y} & \cdots & \frac{v_d}{\lambda_Y} \end{pmatrix},$$

such that the corresponding entropy satisfies

$$H_Y = -\sum_{i=1}^d \pi_{Y,i} \sum_{j=1}^d \frac{v_j}{\lambda_Y} \log\left(\frac{v_j}{\lambda_Y}\right) = -\sum_{j=1}^d s_j \log s_j,$$

for $s_j = \frac{v_j}{\lambda_Y}$. But then, taking $A_Y(\delta) = A_Y^{1+\delta}$, we also get $H_Y(\delta) \approx -\sigma_Y^2 \delta$, for $\delta \in \mathbb{R}$ small, and where $\sigma_Y^2 > 0$. This readily implies $\Delta H_Y \Delta H > 0$, and the proposition immediately follows since $\Delta H_Y \Delta T < 0$.

B.3 Proof of Proposition 2

This follows from Proposition 1 after noticing that for $\delta \in \mathbb{R}$ small, the eigenvector $v(\delta)$ corresponding to the perturbed matrix $A^{1+\delta}$ satisfies

$$v\left(\delta/2\right) \approx \frac{v(\delta) + v(0)}{2}.$$

This implies that $T(\frac{v(\delta)+v(0)}{2}) - T(0)$ has the same sign as ΔT and the opposite sign as $E(v(\delta), v(0))$. Hence $(T(\frac{v(\delta)+v(0)}{2}) - T(0))\Delta H > 0$ and $E(v(\delta), v(0))\Delta H < 0$.

B.4 Proof of Theorem 4

We here povide a sketch of the main steps, and refer the reader to Demetrius et al. [40], Section 3, for more details.

We need to show that $\Delta H \Delta \mathcal{R} > 0$. We first recall the definition of our robustness measure $\mathcal{R}$. Define the probability that the sample mean differs from the value Φ by more than ϵ,

$$P_\epsilon(n) = \mu\left\{x \in \Omega : \left|\frac{1}{n} S_n \varphi(x) - \Phi\right| > \epsilon\right\},$$

where the sample mean is given by,

$$\begin{aligned} S_n\varphi(x) &= \sum_{j=0}^{n-1} \varphi(\tau^j x) \\ &= \log a_{x_0 x_1} + \log a_{x_1 x_2} + \ldots + \log a_{x_{n-1} x_n} \\ &= \log a_{x_0 x_1} a_{x_1 x_2} \cdots a_{x_{n-1} x_n}, \end{aligned}$$

and is such that $\lim_{n\to\infty} S_n\varphi(x) = \int \varphi d\mu = \Phi$.

By the ergodic theorem, $\lim_{n\to\infty} P_\epsilon(n) = 0$; moreover, the convergence rate is at least exponentially fast, so that there exist constants, $c_0, c_1 > 0$, such that,

$$\mu \left\{ x \in \Omega : \left| \frac{1}{n} S_n \varphi(x) - \Phi \right| > \epsilon \right\} \leq c_0 \exp^{-c_1 n} .$$

This motivates the robustness measure given by the fluctuation decay rate,

$$\mathcal{R} \equiv \mathcal{R}_\epsilon = - \lim_{n \to \infty} \left[\frac{1}{n} \log P_\epsilon(n) \right] ,$$

which characterizes the asymptotic value of the probability of the set of trajectories that deviate from the typical trajectory by ϵ or less.

In order to better characterize $\mathcal{R}$, consider the more general decay measures,

$$\overline{\mathcal{R}}(\varphi, E) = -\frac{1}{n} \limsup_{n \to \infty} \left[\mu \left\{ x \in \Omega : \frac{1}{n} \sum_{j=0}^{n-1} \varphi(\tau^j x) \in E \right\} \right]$$

and

$$\underline{\mathcal{R}}(\varphi, E) = -\frac{1}{n} \liminf_{n \to \infty} \left[\mu \left\{ x \in \Omega : \frac{1}{n} \sum_{j=0}^{n-1} \varphi(\tau^j x) \in E \right\} \right] ,$$

where E stands for arbitrary subsets of the real line. (Later we will be interested in sets of the form $E = \{s \in \mathbb{R} : |s - \Phi| > \epsilon\}$.)

Next, one defines the function

$$k_\varphi(s) = r - s - \sup_\nu \left\{ H_\nu(\tau) : \nu \text{ invariant under } \tau \text{ and } \int \varphi d\nu = s \right\} .$$

Then we have,

$$\overline{\mathcal{R}}(\varphi, E) \geq -\inf\{k_\varphi(s) : s \in E\} \text{ for every open set } E,$$

and

$$\underline{\mathcal{R}}(\varphi, E) \leq -\inf\{k_\varphi(s) : s \in E\} \text{ for every closed set } E.$$

Moreover, $k_\varphi(s)$ is continuous and satisfies $k_\varphi(\Phi) = 0$ by the variational principle of equation (19). Hence,

$$\mathcal{R} = \underline{\mathcal{R}}(\varphi, E) = \overline{\mathcal{R}}(\varphi, E) = -\inf\{k_\varphi(s) : s \in E\} = -\min\{k_\varphi(\Phi - \epsilon), k_\varphi(\Phi + \epsilon)\},$$

and actually attains its minimum.

Now consider perturbations of the form $A^{1+\delta}$ corresponding to $\varphi(\delta) = (1 + \delta)\varphi$, where again, $\varphi(x) = \log a_{x_0 x_1}$. One can then define $\mathcal{R}(\delta)$ using $\varphi(\delta)$ instead of φ and show that

$$\mathcal{R}(\delta) = -\min\{k_{\varphi(\delta)}(\Phi(\delta) - (1 + \delta)\epsilon), k_{\varphi(\delta)}(\Phi(\delta) + (1 + \delta)\epsilon)\},$$

where $\lim_{n \to \infty} S_n \varphi(\delta)(x) = \int \varphi(\delta) d\mu(\delta) \equiv \Phi(\delta)$. Also, $H(\delta) = H_{\mu(\delta)}(\tau)$, where $\mu(\delta)$ is the measure corresponding to $\varphi(\delta)$.

Finally, one shows,

$$k_{\varphi(\delta)}(\Phi(\delta)-(1+\delta)\epsilon)=H(\delta)+(1+\delta)\epsilon-\sup\left\{H_\nu(\tau):\nu \text{ invariant under } \tau \text{ and } \int\varphi d\nu=\Phi-\epsilon\right\}$$

and, similarly,

$$k_{\varphi(\delta)}(\Phi(\delta)+(1+\delta)\epsilon)=H(\delta)-(1+\delta)\epsilon-\sup\left\{H_\nu(\tau):\nu \text{ invariant under } \tau \text{ and } \int\varphi d\nu=\Phi+\epsilon\right\}.$$

This readily implies that, from $\Delta\mathcal{R}=\mathcal{R}(\delta)-\mathcal{R}$ and $\Delta H=H(\delta)-H$, we have,

$$\Delta H-\delta\epsilon\leq\Delta\mathcal{R}\leq\Delta H+\delta\epsilon,$$

and hence (for ΔH bounded away from zero) we obtain $\Delta H\Delta\mathcal{R}>0$, which completes the proof.